Multiscale computational modeling of cardiac action potentials

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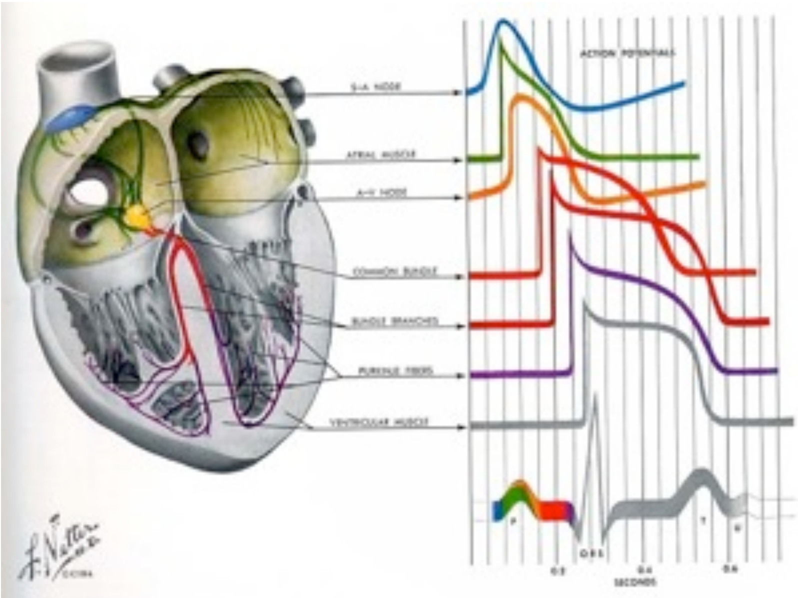
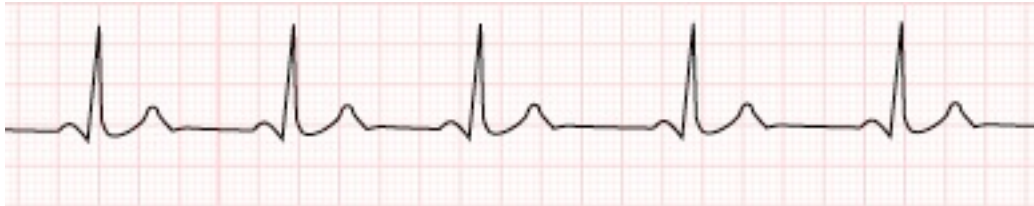
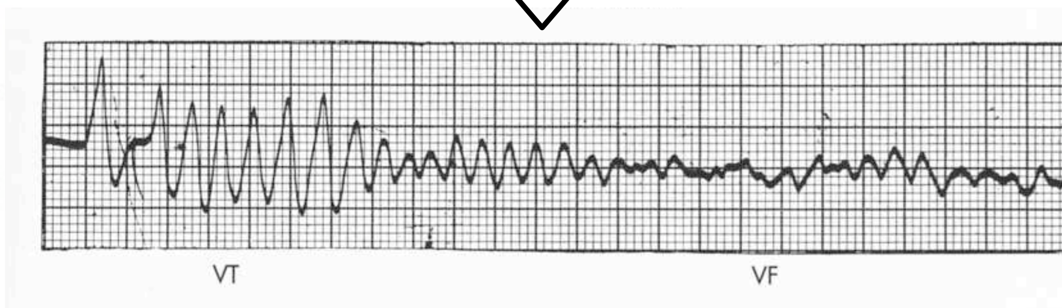
Cardiac arrhythmias





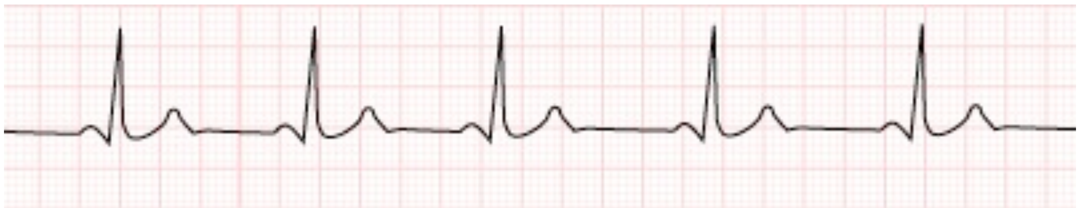


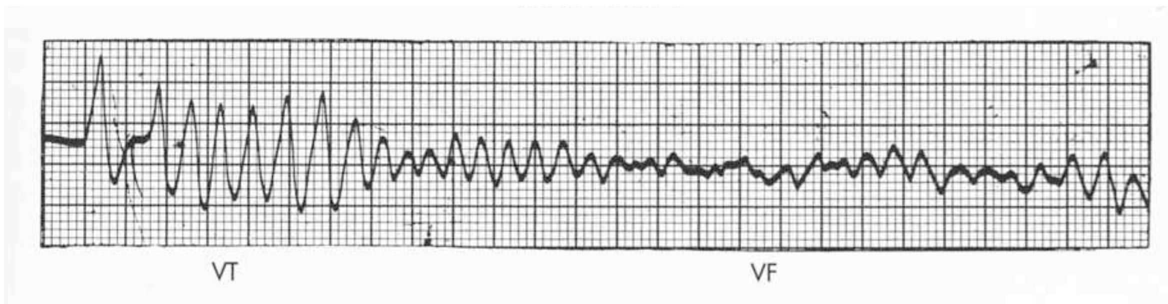
Sudden cardiac death:
~300,000 deaths/year

Ventricular tachycardia
- Rapid activation
- May impair pumping
- May degenerate to VF

Ventricular fibrillation
- Loss of synchronous activation
- Syncope, death

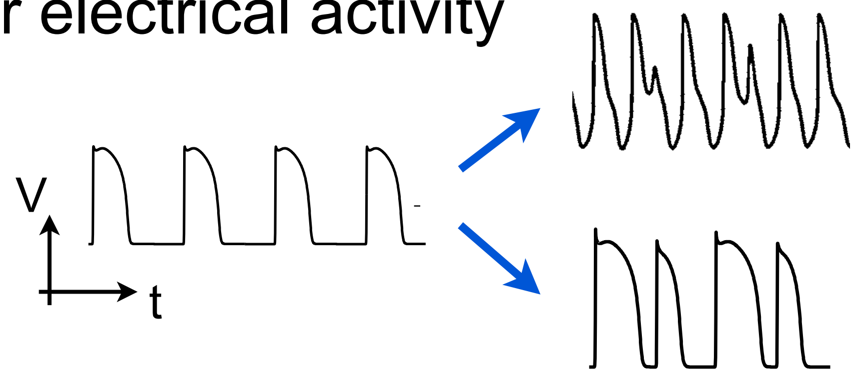
- How do cardiac arrhythmias initiate?
- How are they sustained?
- What can we do to prevent their occurrence?
- How can we terminate them?





Initiation

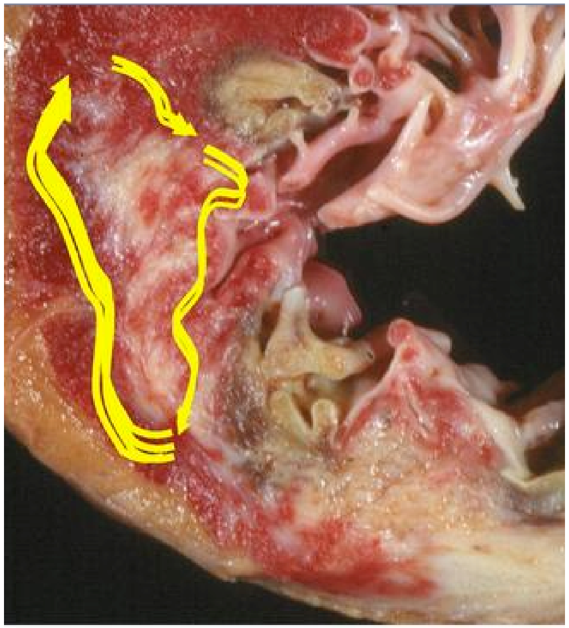
Abnormal cellular electrical activity



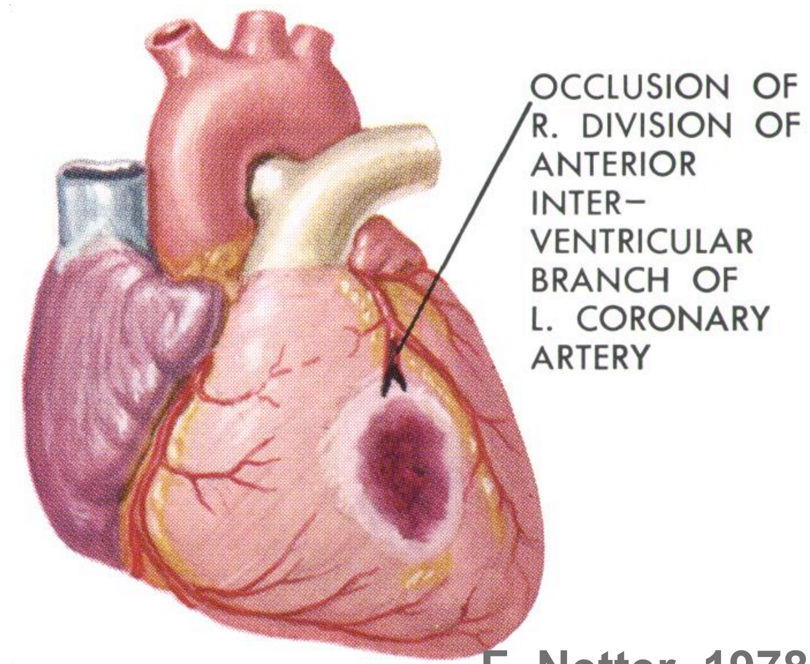
Early afterdepolarization

Repolarization alternans

Structural heterogeneity

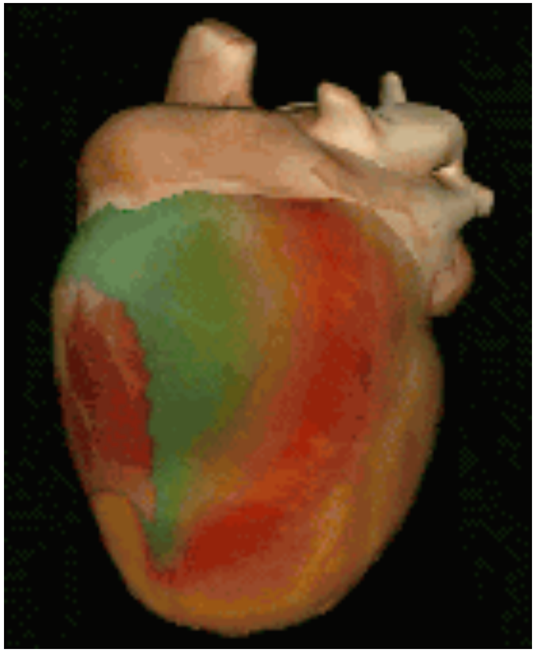


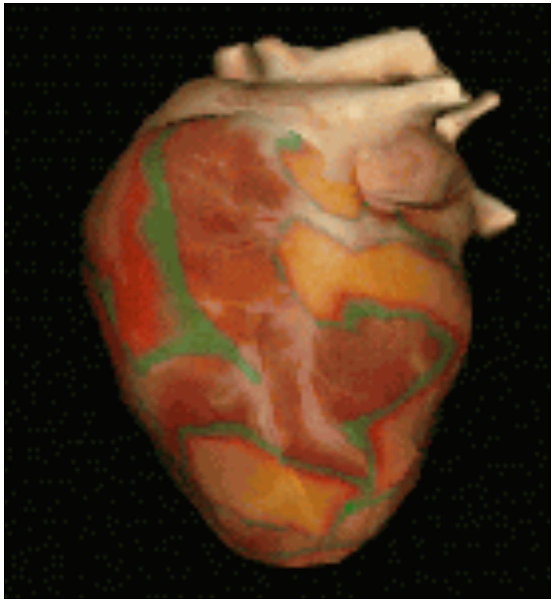
Bill Stevenson, KITP seminar, 2006.

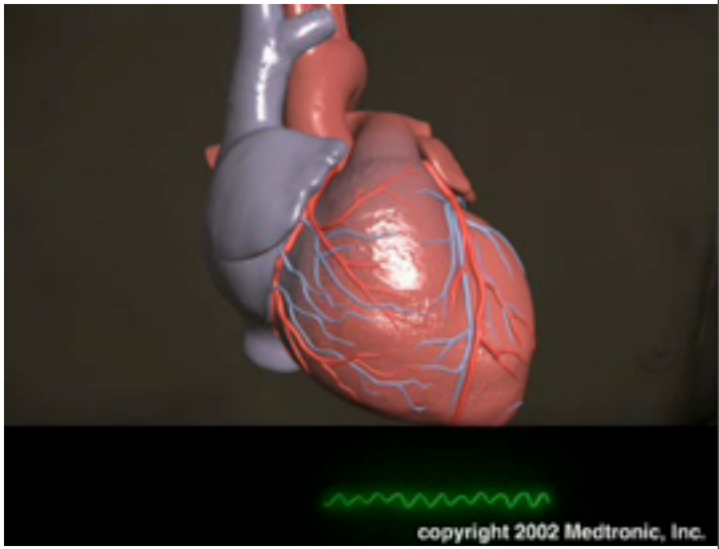


F. Netter, 1978

Cardiac arrhythmia mechanisms



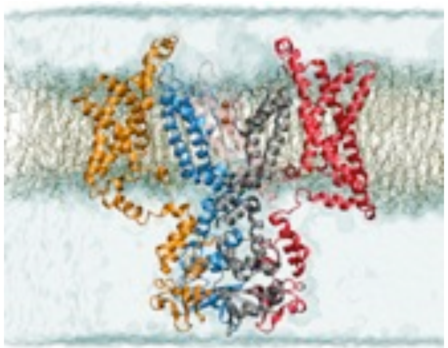


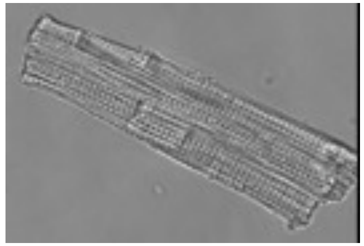


thevirtualheart.org

Multiscale phenomena

single channel → single cell → tissue, organ





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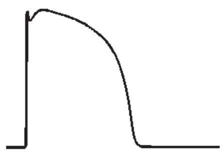
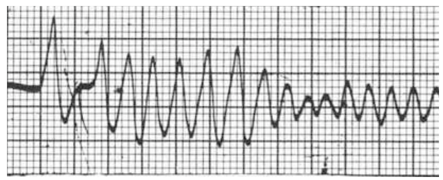
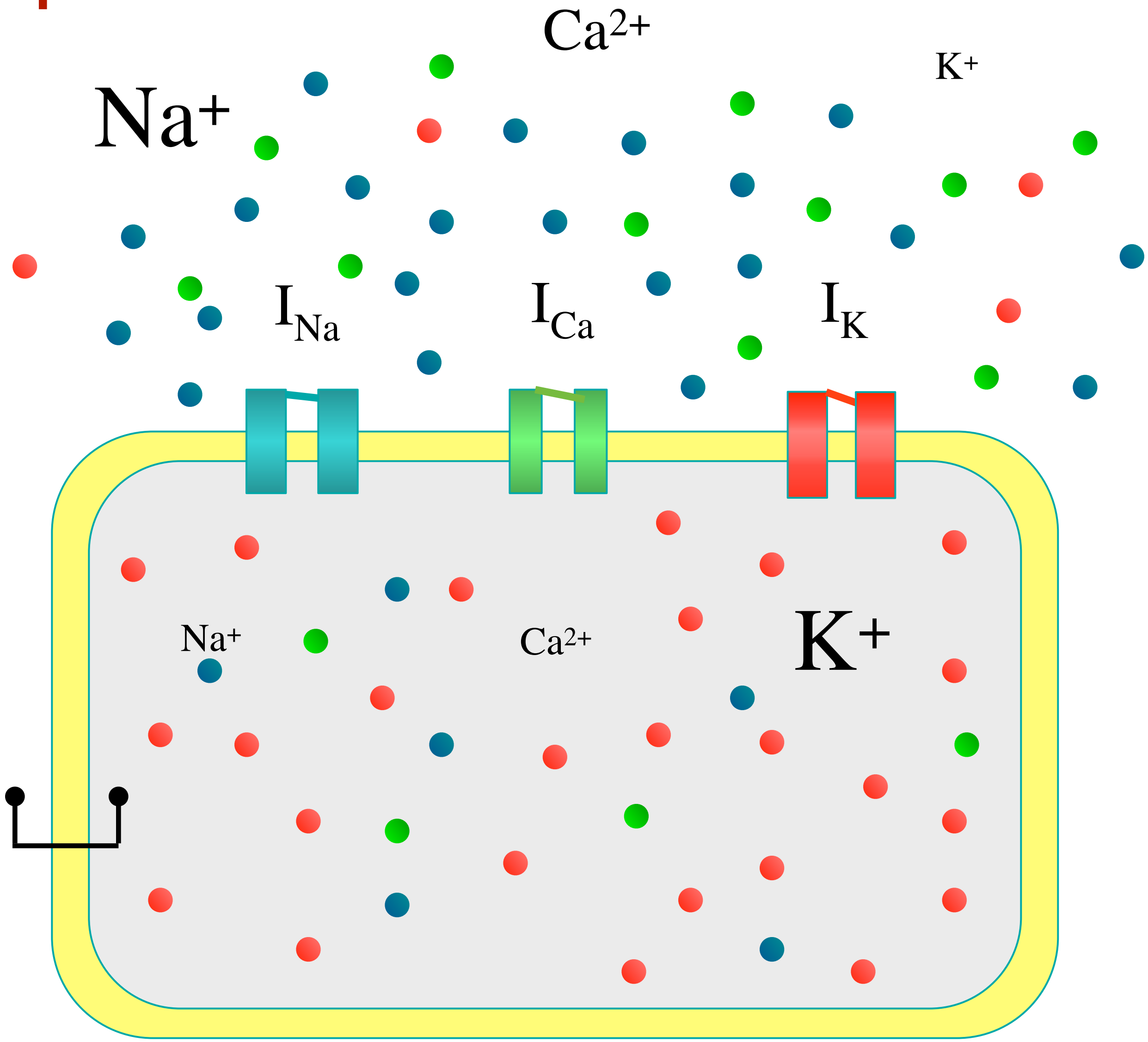




10 µs | 100 µs | 1 ms | 10 ms | 100 ms | 1 s | 10 s | 100 s

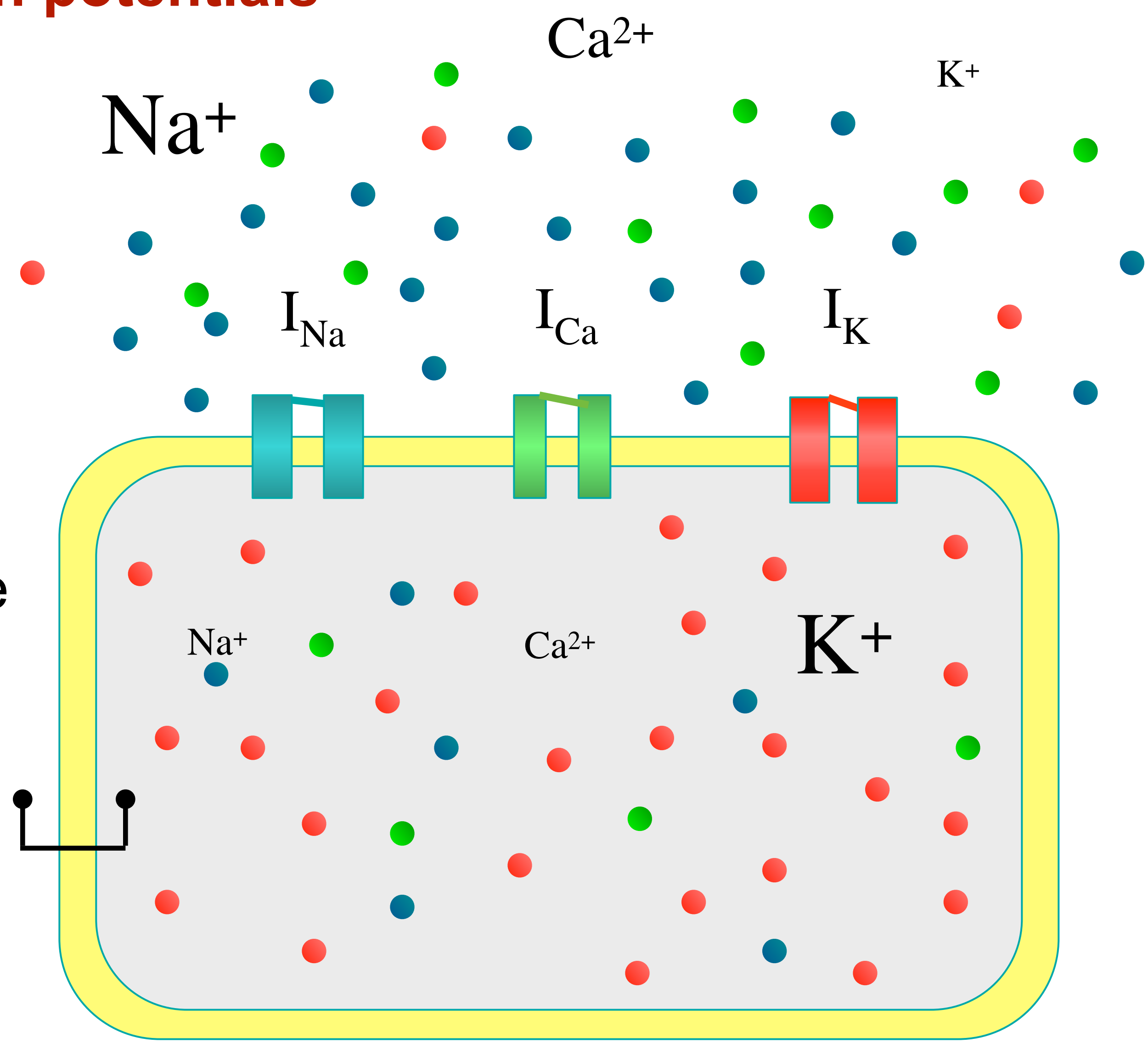
Cardiac action potentials



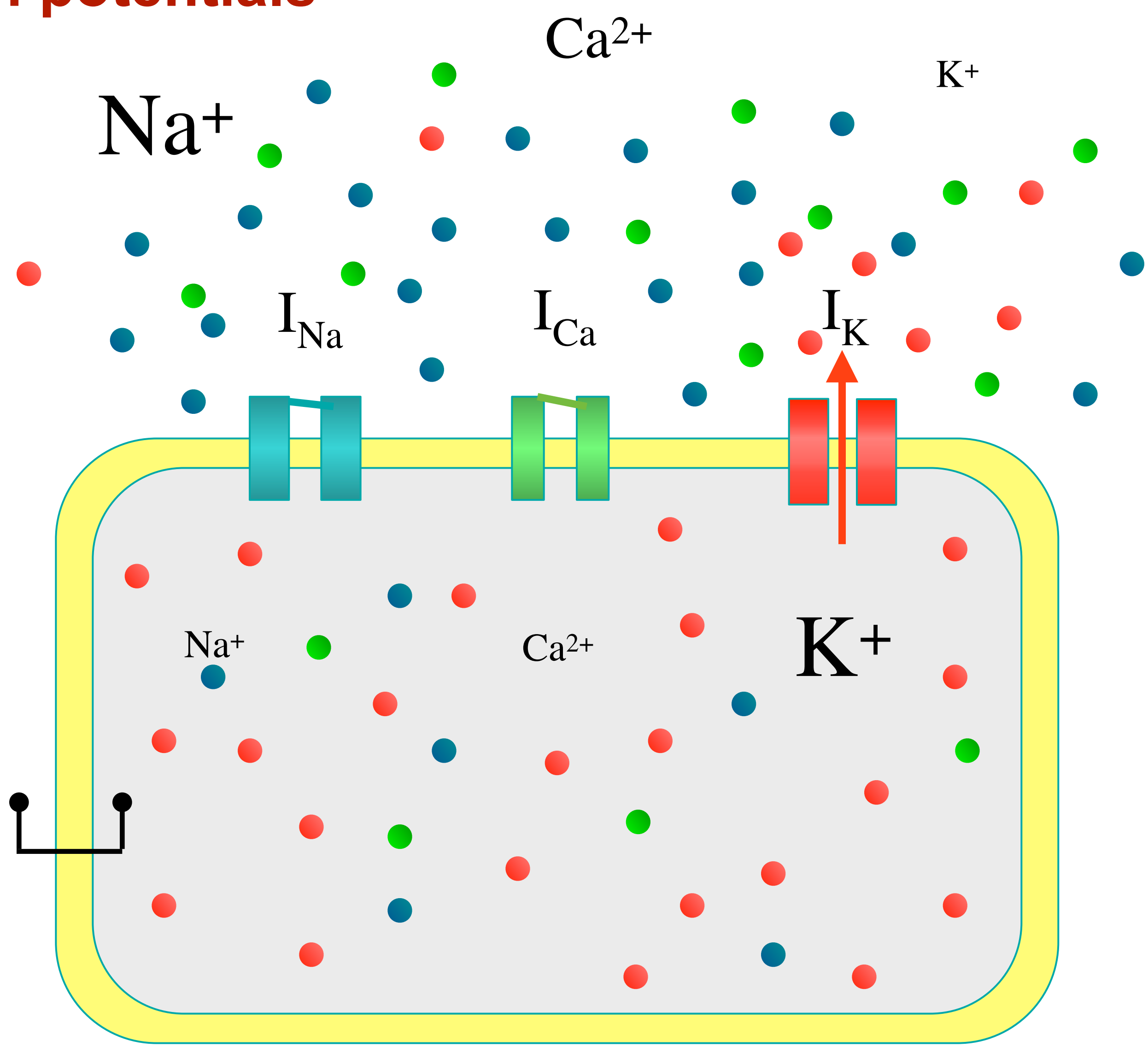
Transmembrane potential:
If ionic distribution neutral,
V= ? mV.

Cardiac action potentials

Transmembrane potential:
If ionic distribution neutral,
V= 0 mV.



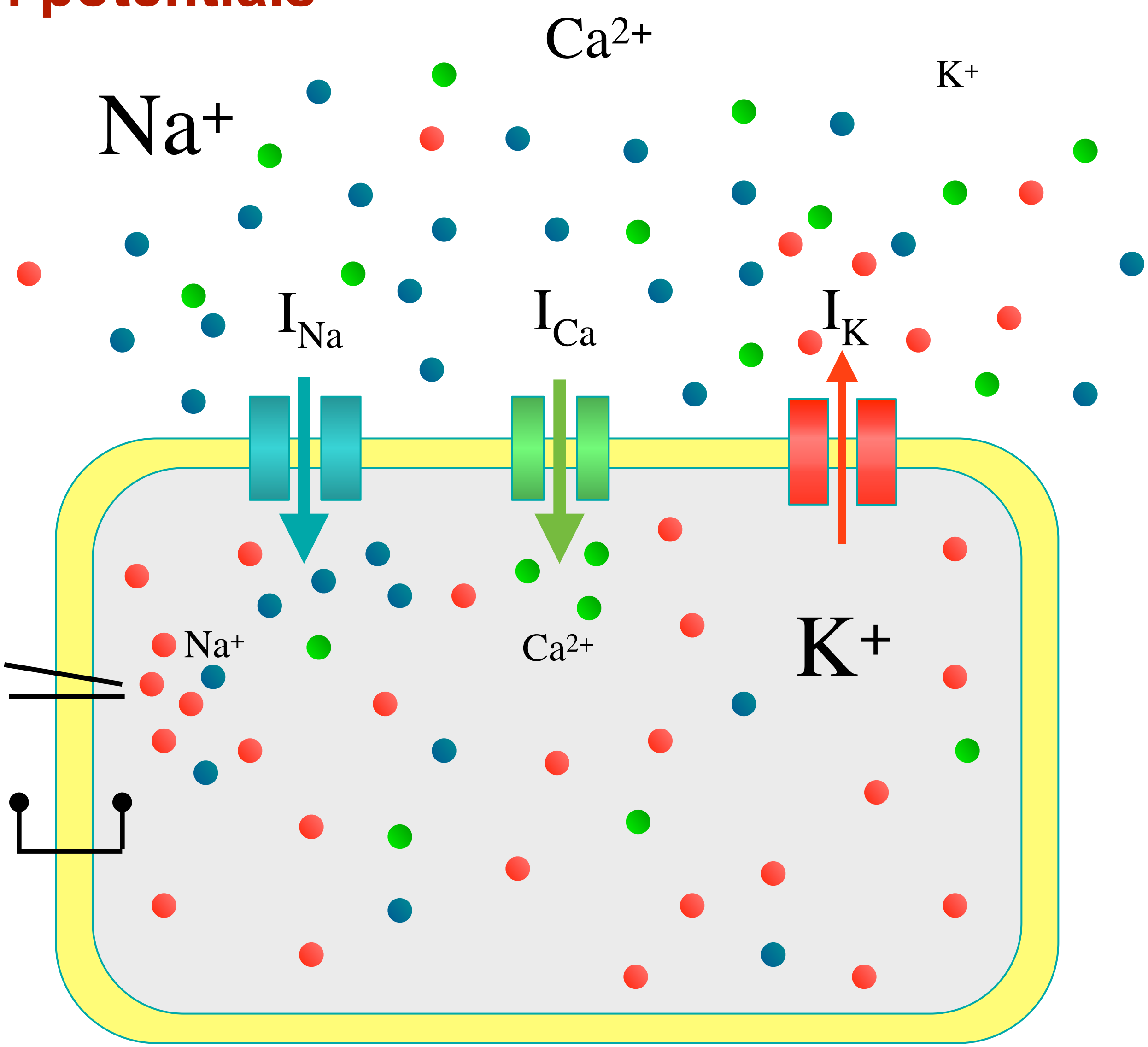
Cardiac action potentials



Cardiac cells rest at V≈−85 mV with some K channels open.

$$E_{\mathrm{K}} = \frac{RT}{zF} \ln \frac{[\mathrm{K}^+]_o}{[\mathrm{K}^+]_i} \approx -85\mathrm{mV}$$

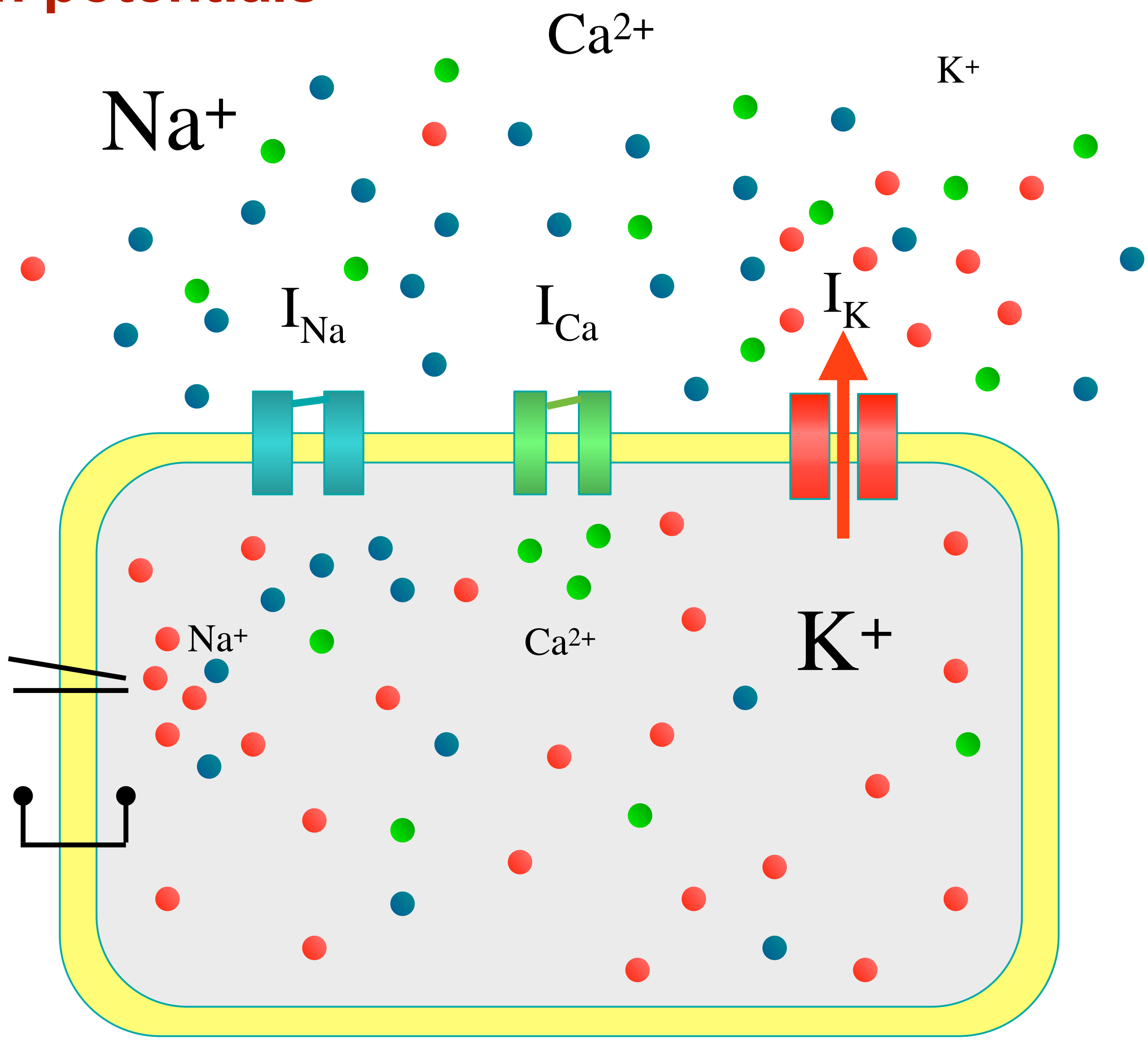
Cardiac action potentials



Injection of a stimulus current initiates depolarization, which cause Na^+ and Ca^+ channels to open and further depolarize the membrane, V≈+20 mV

(E_{Na}≈+50 mV, E_{Ca}≈+30 mV)

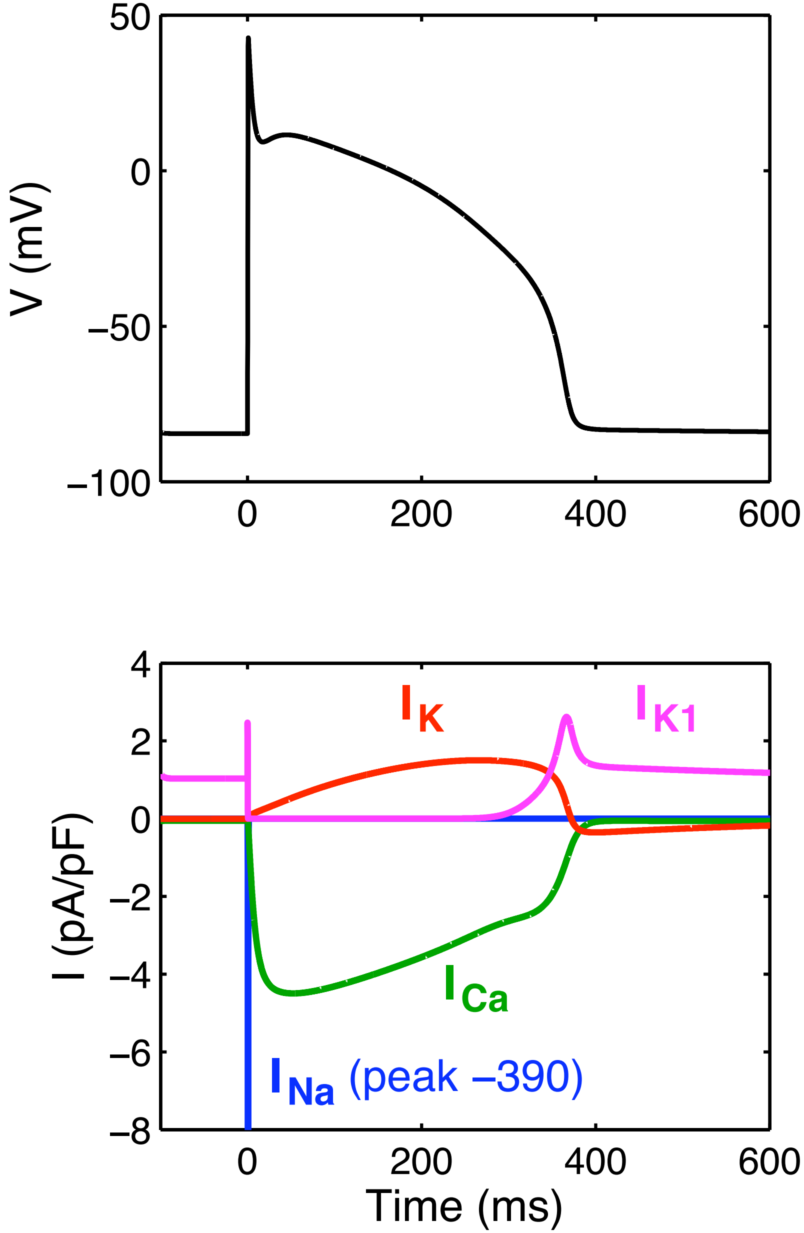
Cardiac action potentials



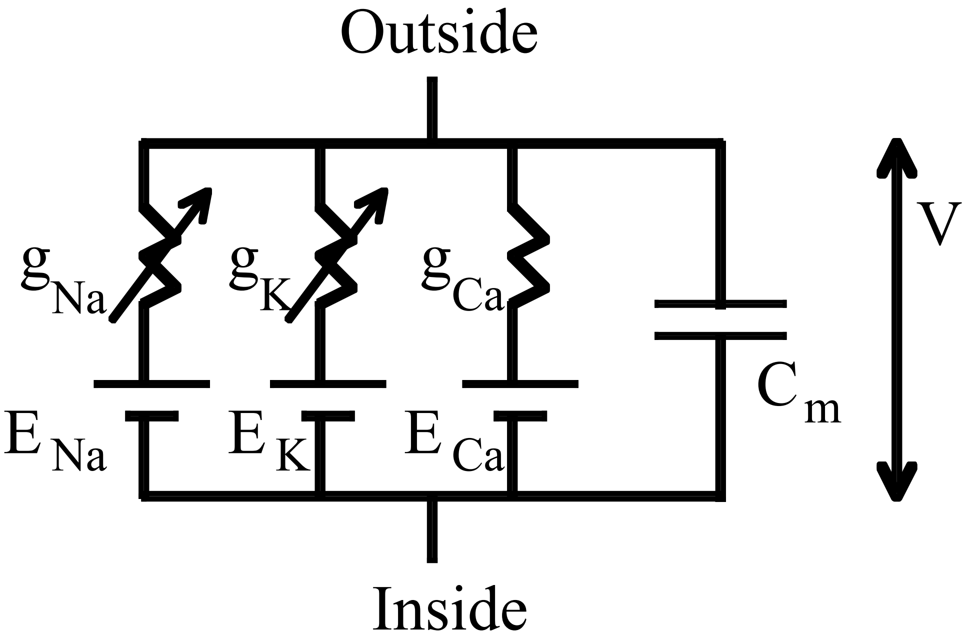
Na+ and Ca+ channels inactivate (close) with prolonged depolarization. K+ channels open and cause repolarization to V≈−85 mV.

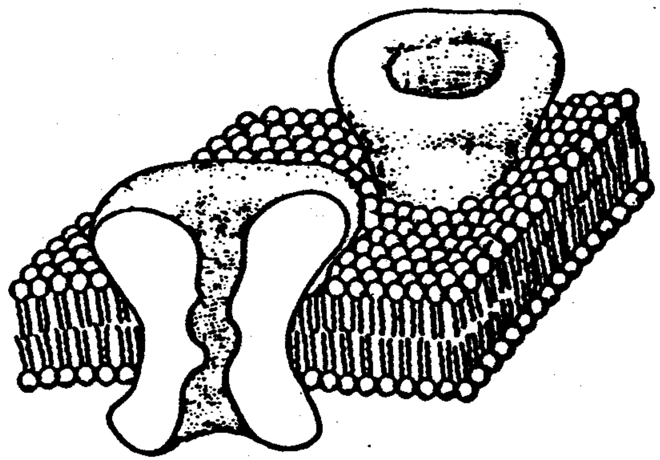
Cardiac action potentials

- Upstroke of ventricular AP is Na^+ mediated.
- A prolonged inward Ca^{2+} current prolongs the AP (plateau).
- Ca^{2+} influx triggers additional Ca^{2+} release from the sarcoplasmic reticulum.
- Cytoplasmic Ca^{2+} produces muscle contraction.
- Cardiac cells have many different types of K^+ channels.



The membrane as an electrical circuit





Equation for capacitor: $Q = C_m V$

Current across capacitor: $I_C = dQ/dt = C_m \, dV/dt$

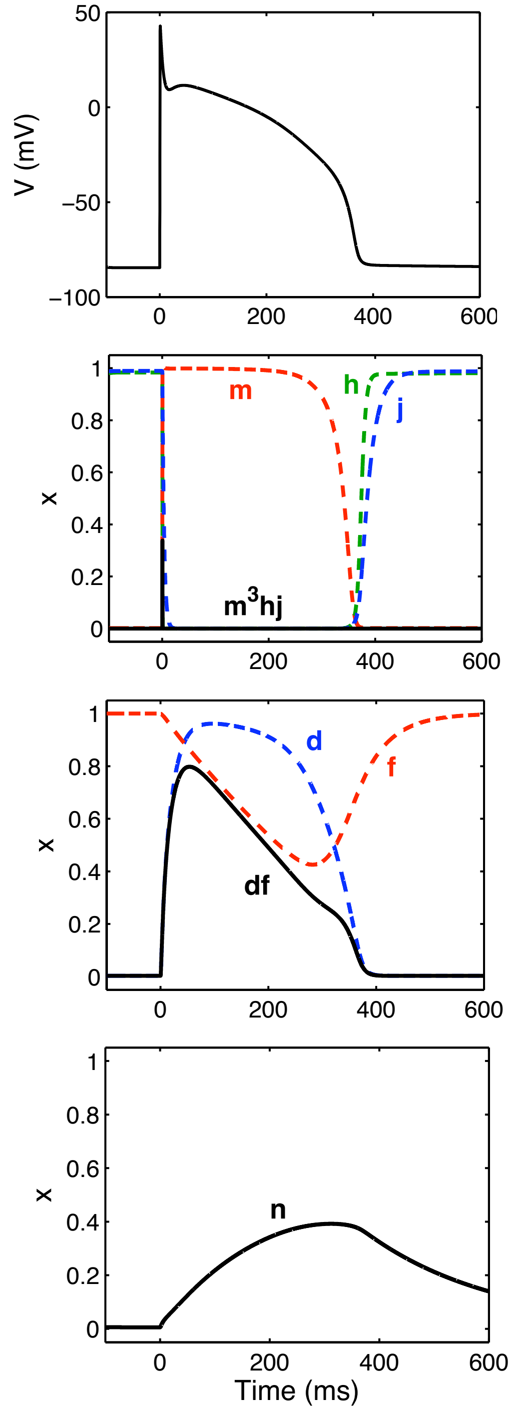
Charge conservation: $I = I_C + I_{ion} = 0$

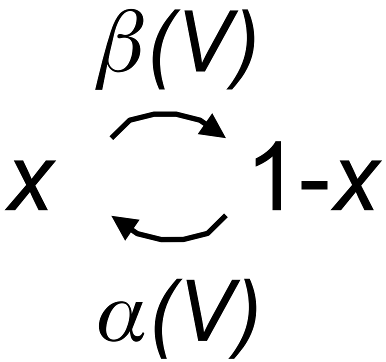
Hence, $\boxed{dV/dt = -I_{ion}/C_m}$ where $I_{ion} = I_{Na} + I_K + I_{Ca}$

Examples of currents with voltage-gated conductances:

$$
\begin{aligned}
I_{Na} &= g_{Na} \cdot m^3 \cdot h \cdot j \cdot (V - E_{Na}) \\
I_{Ca} &= g_{Ca} \cdot d \cdot f \cdot (V - E_{Ca}) \\
I_K &= g_K \cdot n \cdot (V - E_K)
\end{aligned}
$$

m, h, j, d, f, n
represents the fraction of gates that are open





x: fraction of gates that are open

1-x: fraction of gates that are closed

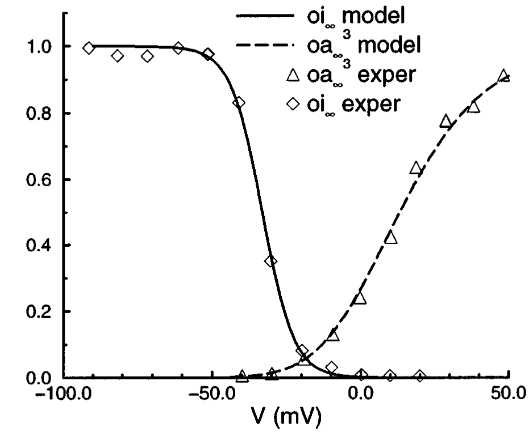
$\alpha(V)$: opening rate

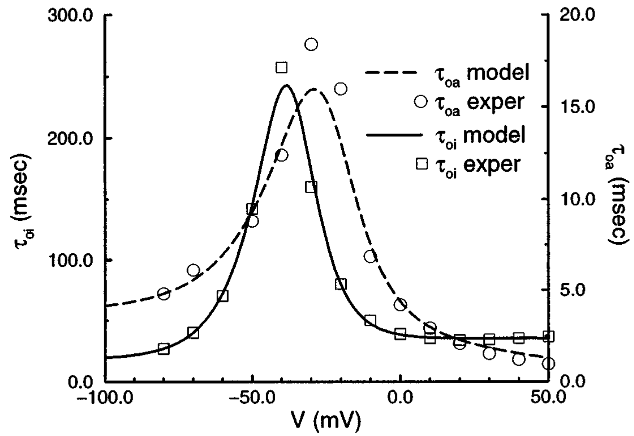
$\beta(V)$: closing rate

ODE for gating variable:

$$
\begin{aligned}
dx/dt &= \alpha_x(1-x) - \beta_x x \\
&= -(\alpha_x + \beta_x)x + \alpha_x \\
&= (x_\infty - x)/\tau_x
\end{aligned}
$$

where
$$x_\infty = \alpha_x/(\alpha_x + \beta_x)$$
$$\tau_x = 1/(\alpha_x + \beta_x).$$





Solution for constant V:

$$dx/dt = (x_\infty - x)/\tau_x$$

$$1/(x_\infty - x)dx = 1/\tau_x dt$$

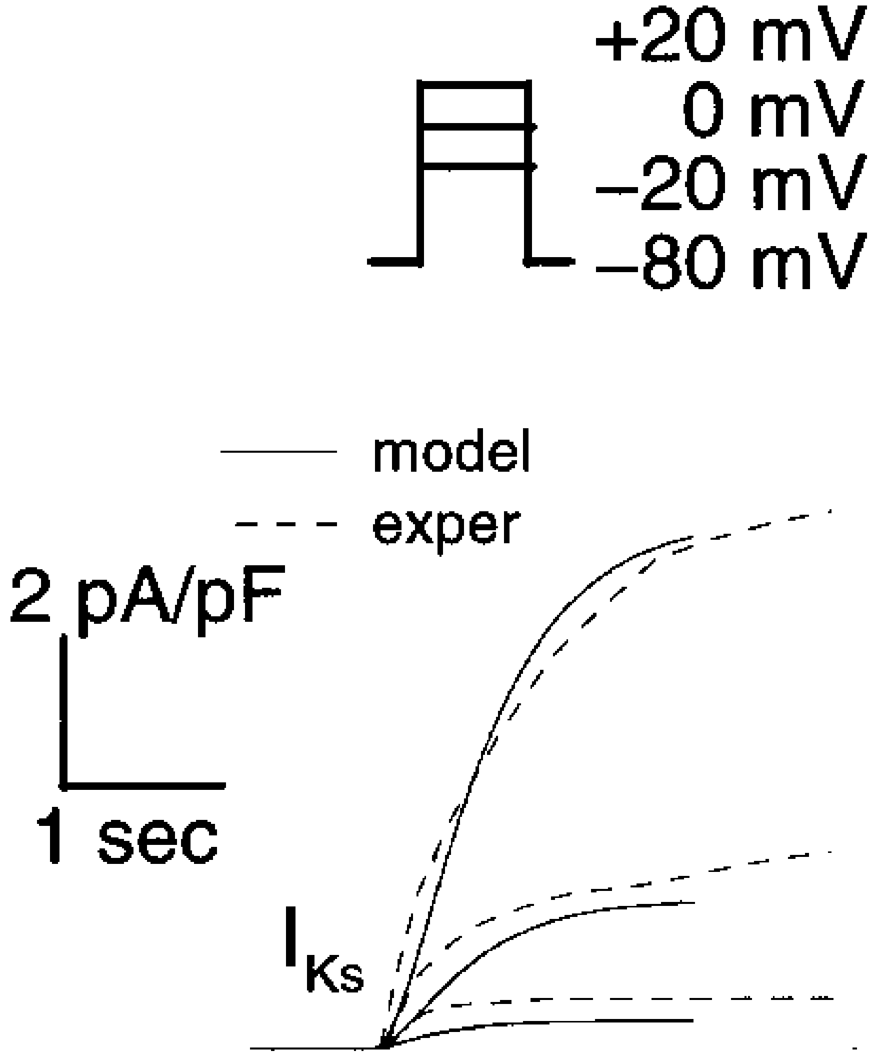
$$\int_{x_0}^{x} 1/(x_\infty - x')dx' = \int_0^t 1/\tau_x dt'$$

$$\left[-\ln(x_\infty - x')\right]_{x_0}^{x} = t/\tau_x$$

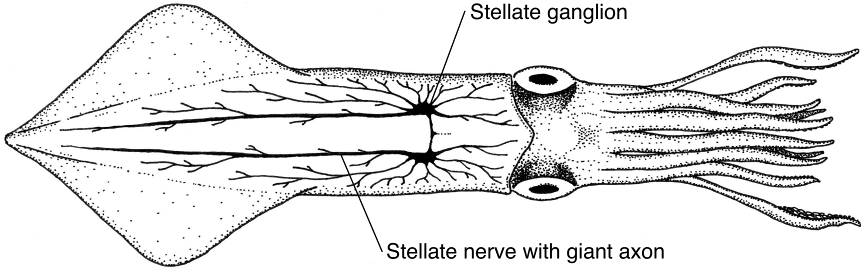
$$\ln\frac{x_\infty - x}{x_\infty - x_0} = -t/\tau_x$$

$$\frac{x_\infty - x}{x_\infty - x_0} = \exp(-t/\tau_x)$$

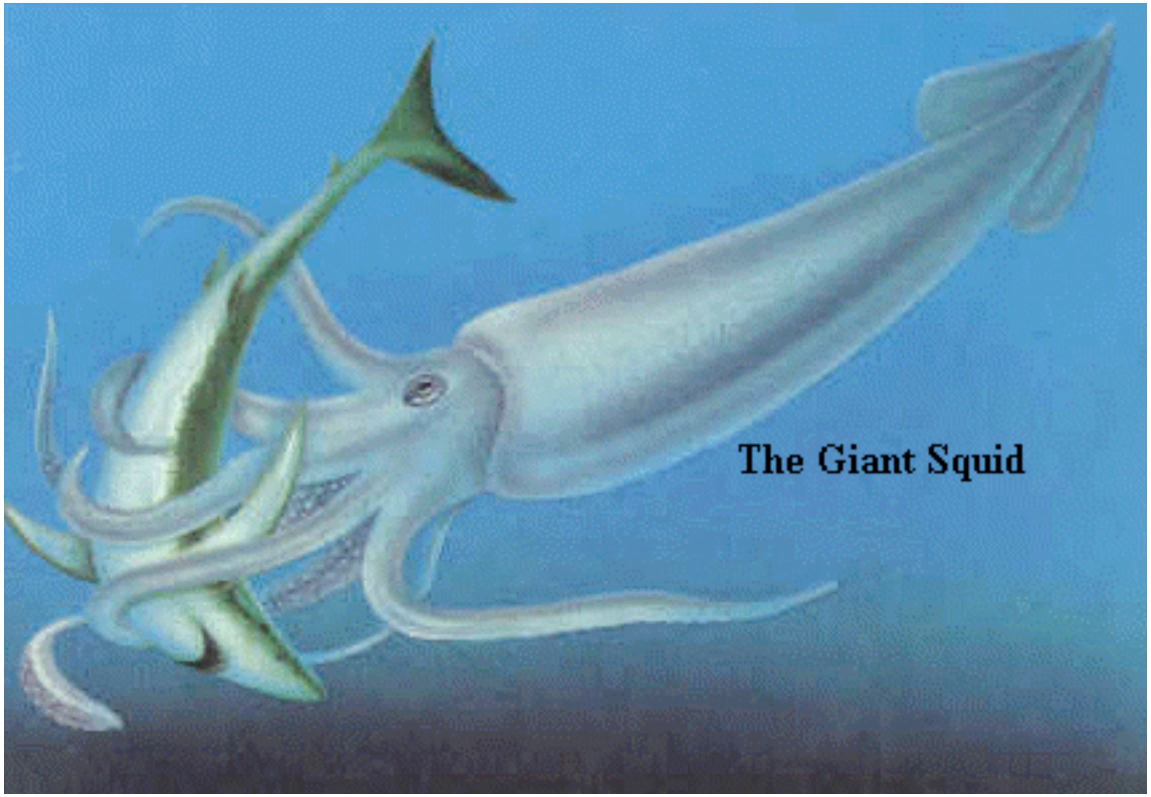
$$x = x_\infty - (x_\infty - x_0)\exp(-t/\tau_x)$$

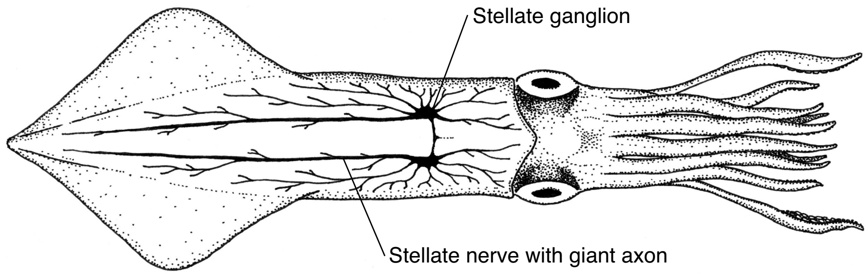


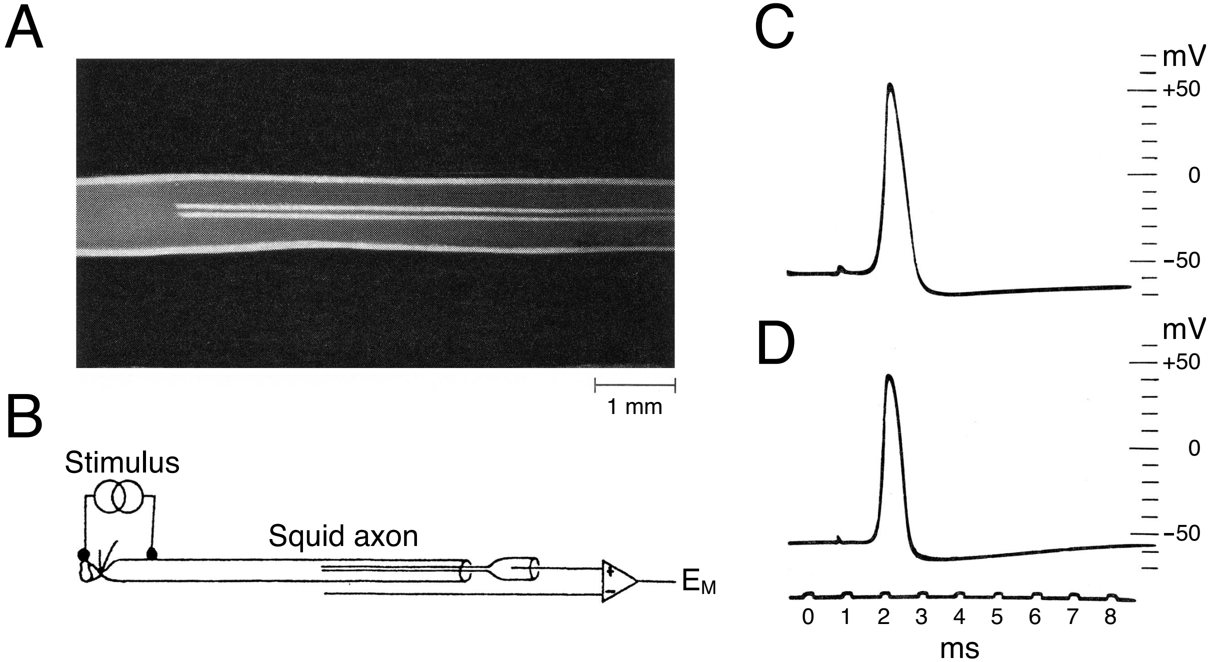
The Hodgkin-Huxley model of the squid giant axon



The axon is giant,
not the squid







Action potential recordings from squid giant axon

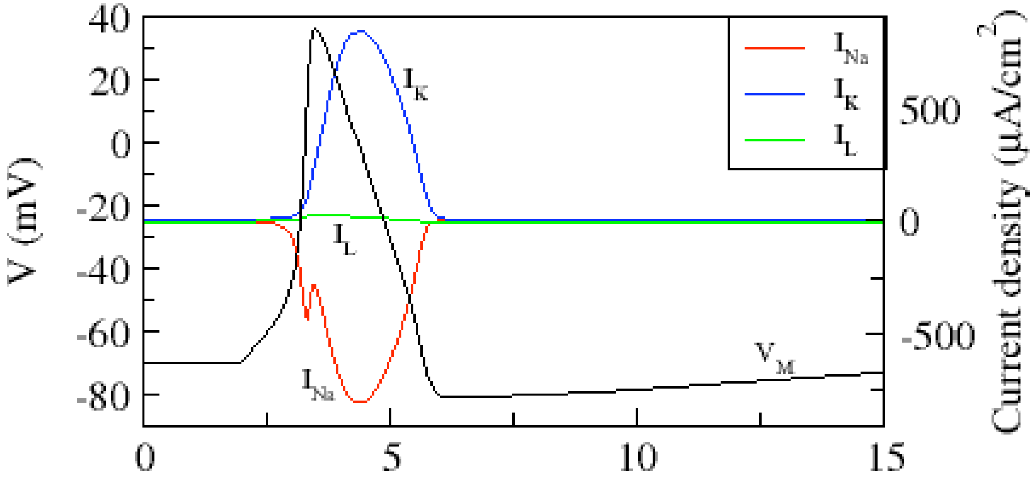
Full Hodgkin-Huxley model

$$\frac{dV}{dt} = -\frac{1}{C}[(\bar{g}_{\mathrm{Na}} m^3 h(V - E_{\mathrm{Na}}) + \bar{g}_{\mathrm{K}} n^4 (V - E_{\mathrm{K}}) + \bar{g}_{\mathrm{L}}(V - E_{\mathrm{L}}) + I_{\mathrm{stim}}],$$

$$\frac{dm}{dt} = \alpha_m(1-m) - \beta_m m,$$

$$\frac{dh}{dt} = \alpha_h(1-h) - \beta_h h,$$

$$\frac{dn}{dt} = \alpha_n(1-n) - \beta_n n,$$



$$\alpha_m = 0.1(V+35)/(1-\exp(-(V+35)/10)),$$

$$\beta_m = 4\exp(-(V+60)/18),$$

$$\alpha_h = 0.07\exp(-(V+60)/20),$$

$$\beta_h = 1/(\exp(-(V+30)/10)+1),$$

$$\alpha_n = 0.01(V+50)/(1-\exp(-(V+50)/10)),$$

$$\beta_n = 0.125\exp(-(V+60)/80).$$

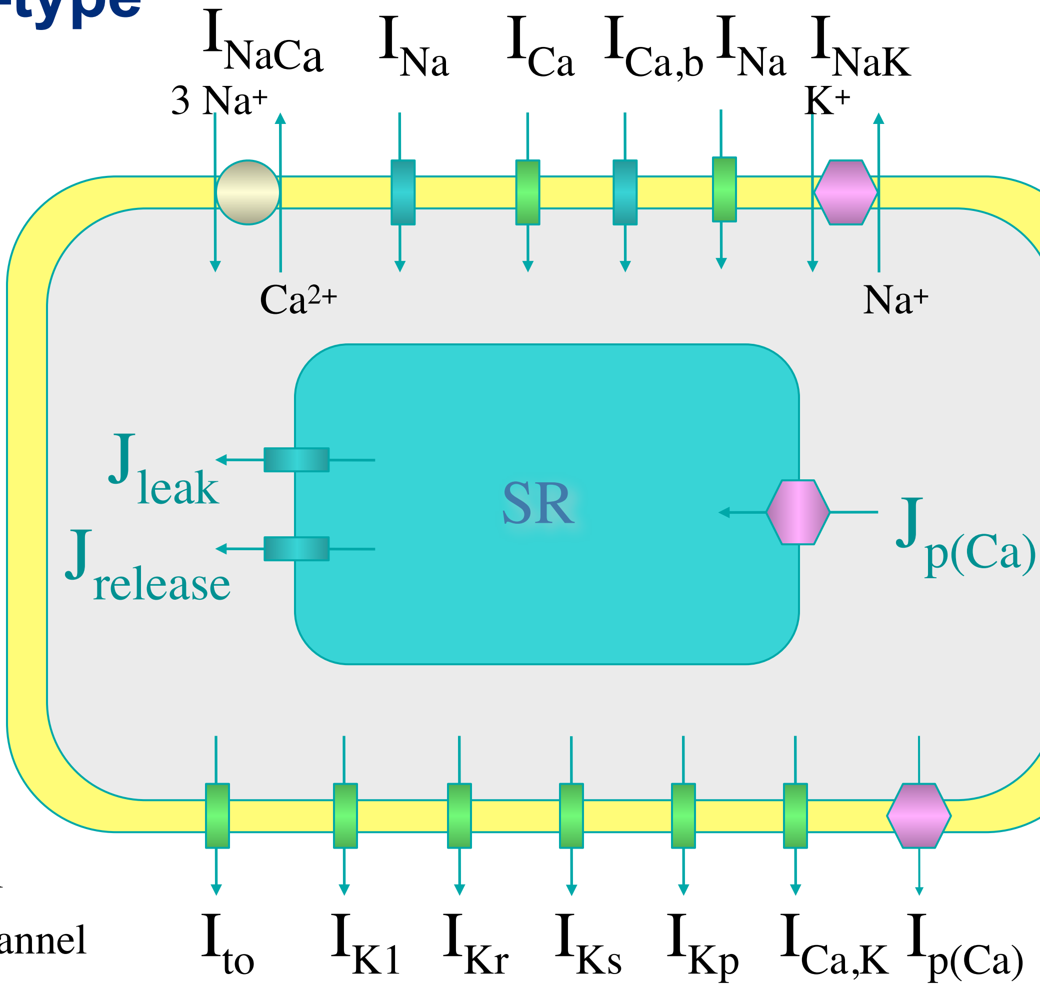
Single cardiac myocyte model example

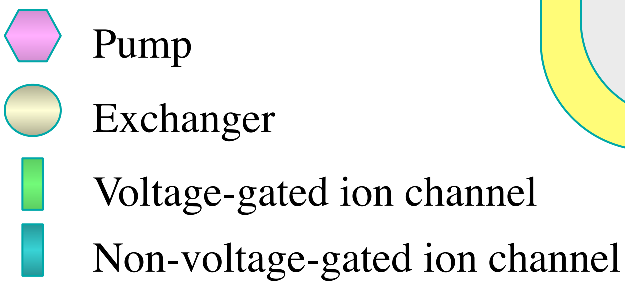
Hodgkin-Huxley-type

$$\frac{dV}{dt} = -\sum I_i / C_m$$

$$I_i = g_i \cdot (V - E_i)$$

$$g_i = f(V,t)$$





CVM model of the canine ventricular myocyte
13 state variables and ~60 parameters

courtesy of R. Gilmour

Single cardiac myocyte model example

Hodgkin-Huxley-type

$$\frac{dV}{dt} = -\sum I_i / C_m$$

$$I_i = g_i \cdot (V - E_i)$$

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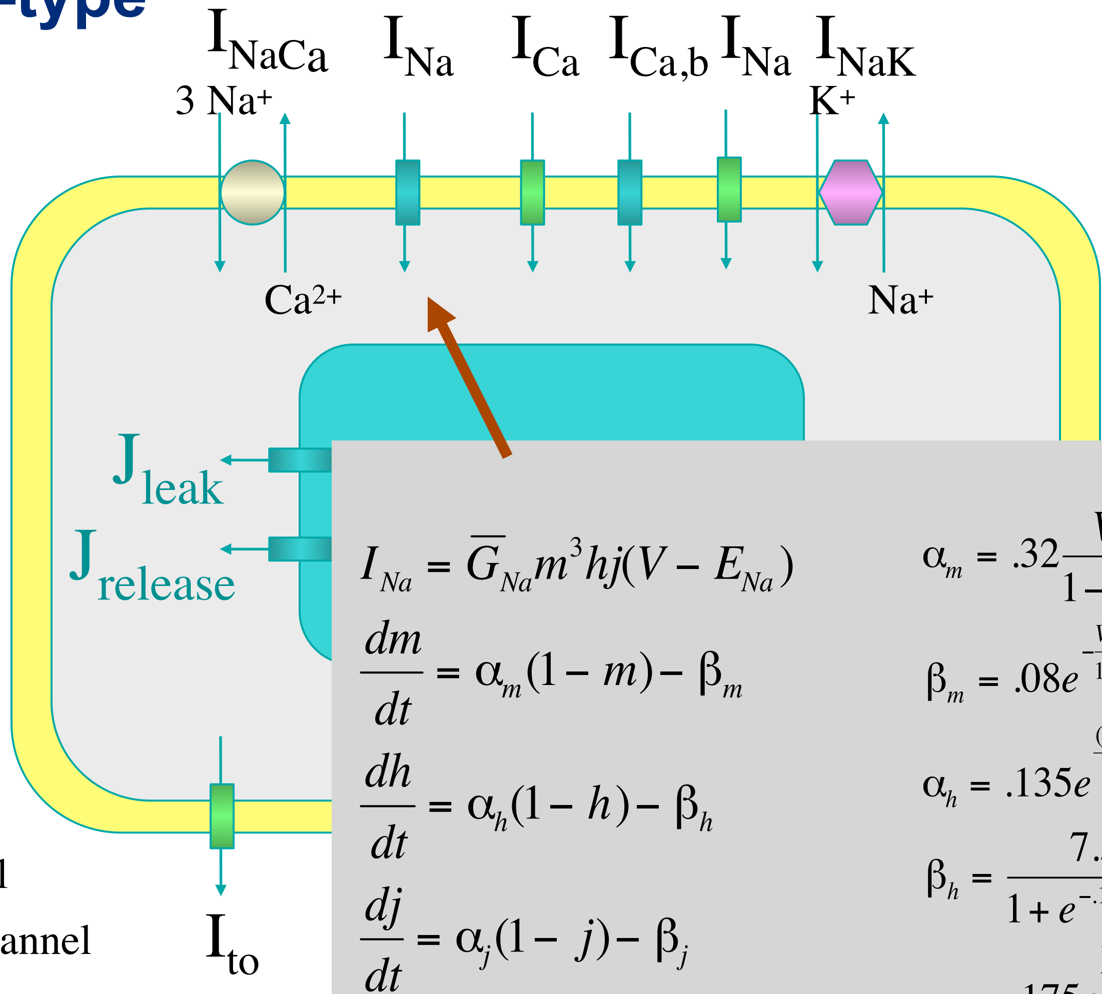


Pump

Exchanger

Voltage-gated ion channel

Non-voltage-gated ion channel

CVM mod

~13 state v

$$I_{Na} = \overline{G}_{Na} m^3 hj(V - E_{Na})$$

$$\frac{dm}{dt} = \alpha_m(1 - m) - \beta_m$$

$$\frac{dh}{dt} = \alpha_h(1 - h) - \beta_h$$

$$\frac{dj}{dt} = \alpha_j(1 - j) - \beta_j$$

$$E_{Na} = \frac{RT}{F} \ln\left(\frac{[Na^+]_o}{[Na^+]_i}\right)$$

$$\alpha_m = .32 \frac{V + 47.13}{1 - e^{-.1(V+47.13)}}$$

$$\beta_m = .08 e^{-\frac{V}{11}}$$

$$\alpha_h = .135 e^{\frac{(V+80)}{-6.8}}$$

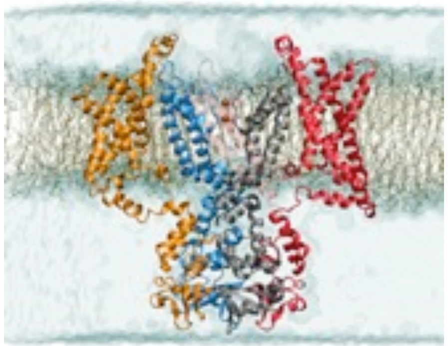
$$\beta_h = \frac{7.5}{1 + e^{-.1(V+11)}}$$

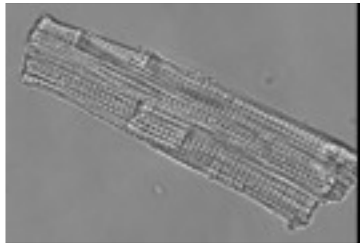
$$\alpha_j = \frac{.175 e^{\frac{V+100}{-23}}}{1 + e^{.15(V+79)}}$$

$$\beta_j = \frac{.3}{1 + e^{-.1(V+32)}}$$

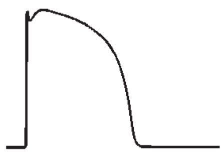
Multiscale phenomena

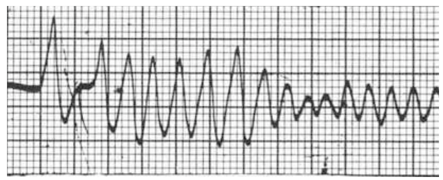
single channel → single cell → tissue, organ





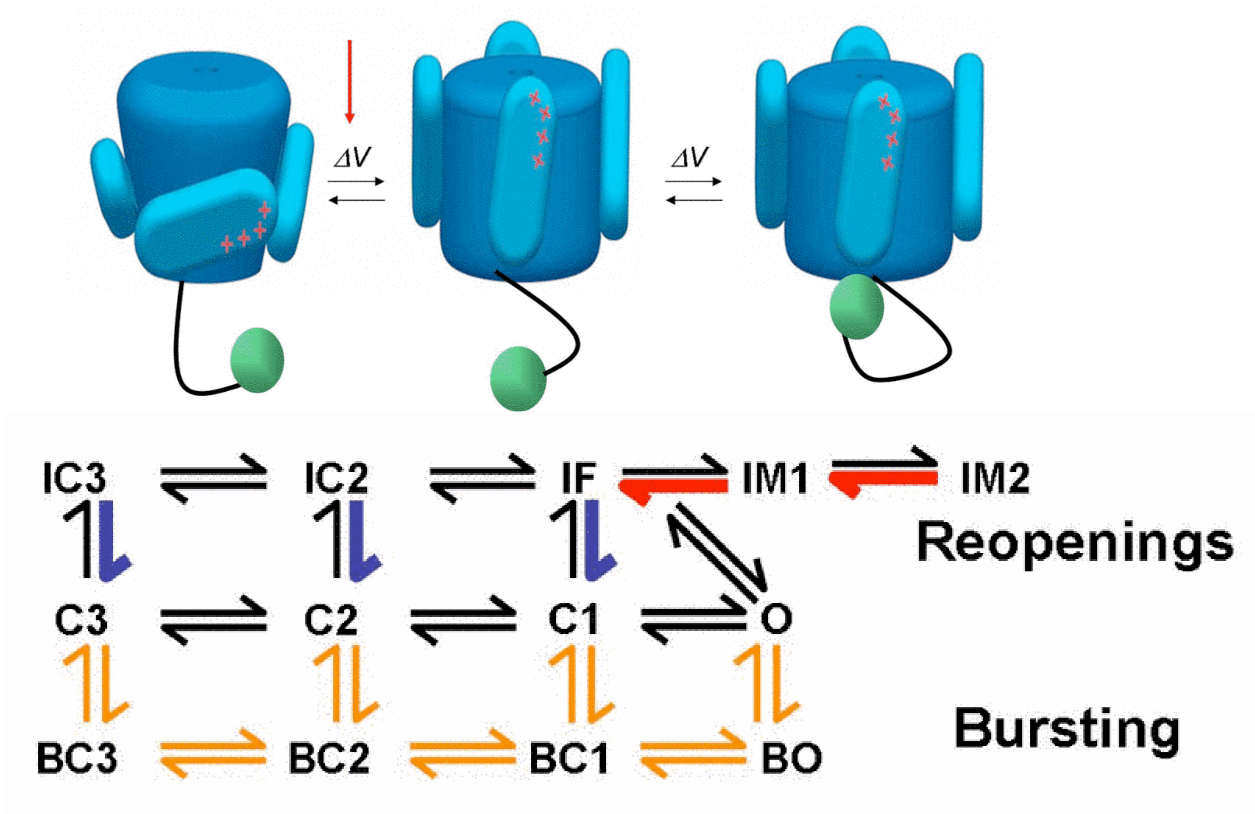
1 Å | 1 nm | 10 nm | 100 nm | 1 µm | 10 µm | 100 µm | 1 mm | 1 cm | 10 cm





10 µs | 100 µs | 1 ms | 10 ms | 100 ms | 1 s | 10 s | 100 s

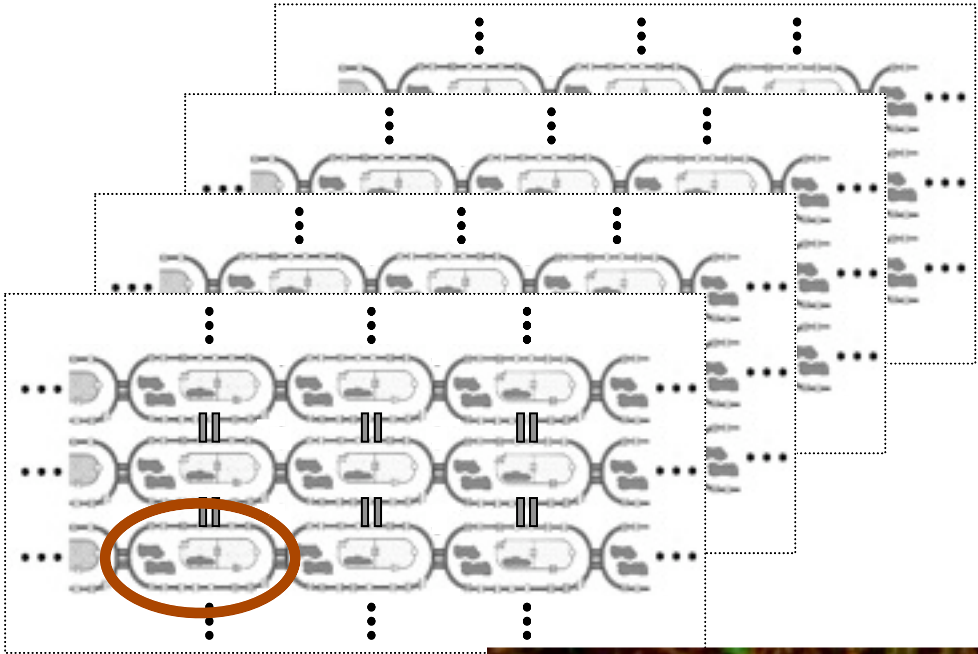
Single-channel modeling: Markov model

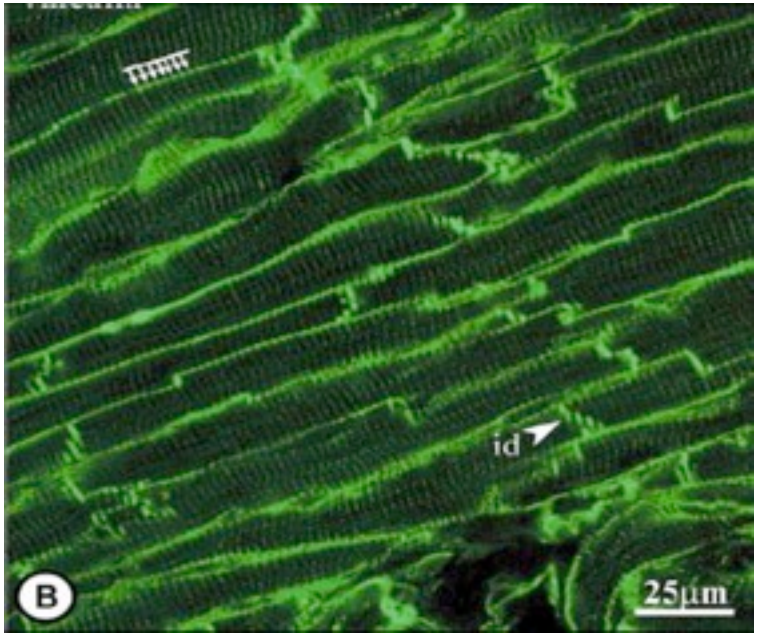


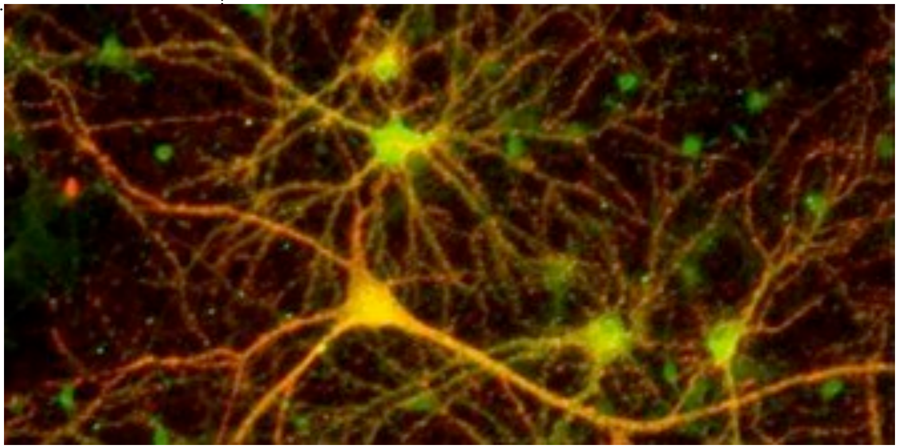
- May be based on channel structure
- Gates not necessarily independent
- May reproduce experimental data better than HH
- Integration time step usually small

Three-dimensional virtual cardiac tissue

Virtual cells coupled by Ohmic resistances (gap junctions)

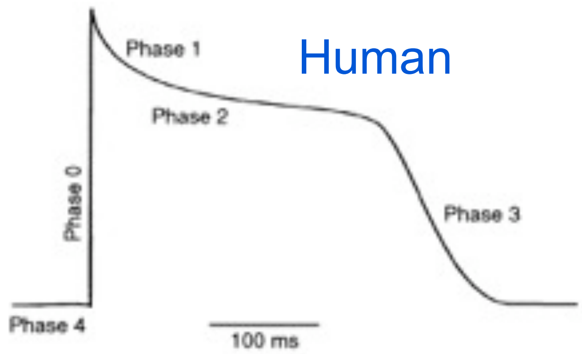


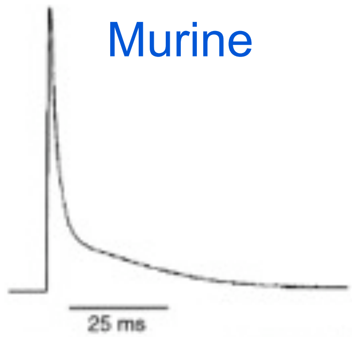




Why use computational modeling for cardiac electrophysiology?

- Rodent cardiac myocytes have fundamentally different channel expression levels (especially repolarizing currents). Therefore, transgenic models are not always appropriate.
- Modeling allows one to monitor each component simultaneously – not possible in experiments.
- Dynamics can be observed at resolutions that are unattainable experimentally or clinically.
- It is often faster and cheaper to do so.

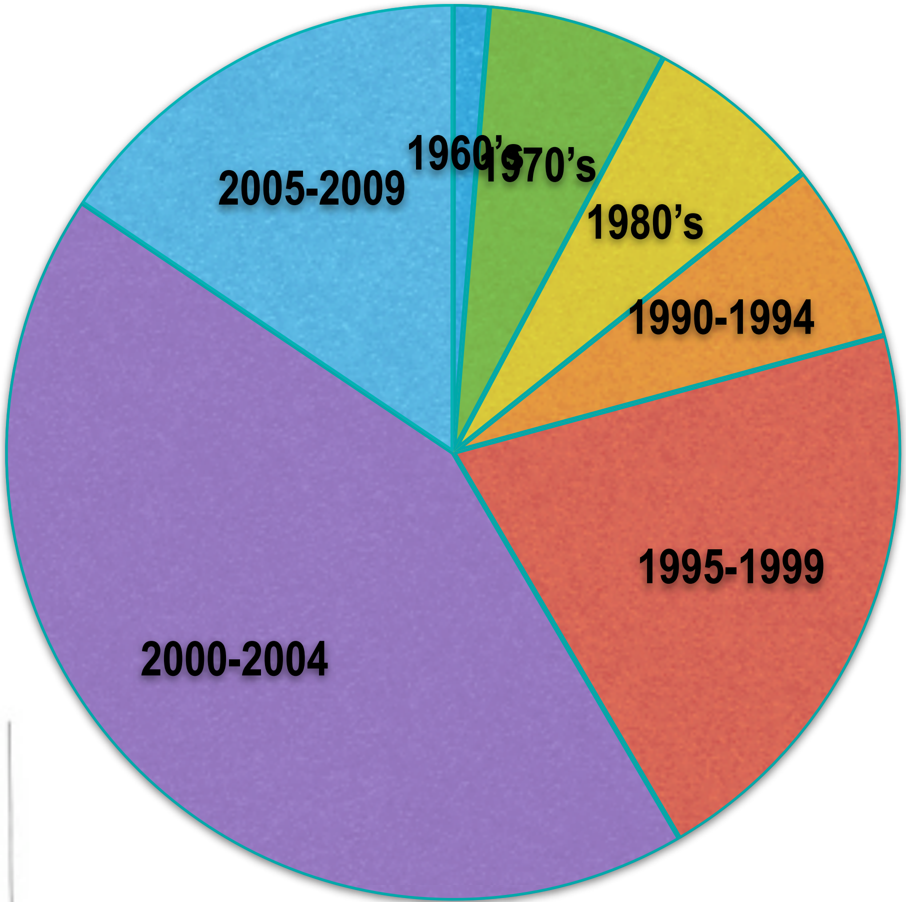


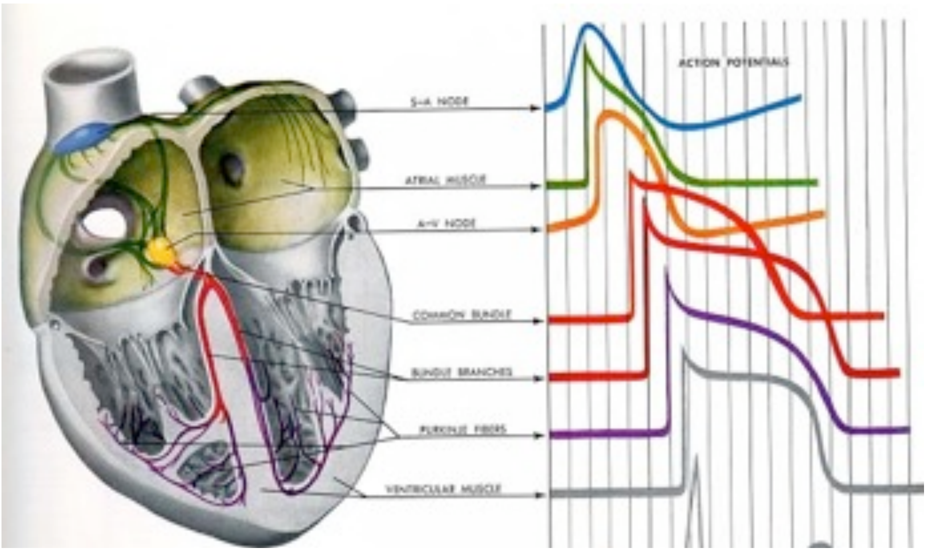


Nerbonne. *Trends Cardiovasc. Med.* 2004.

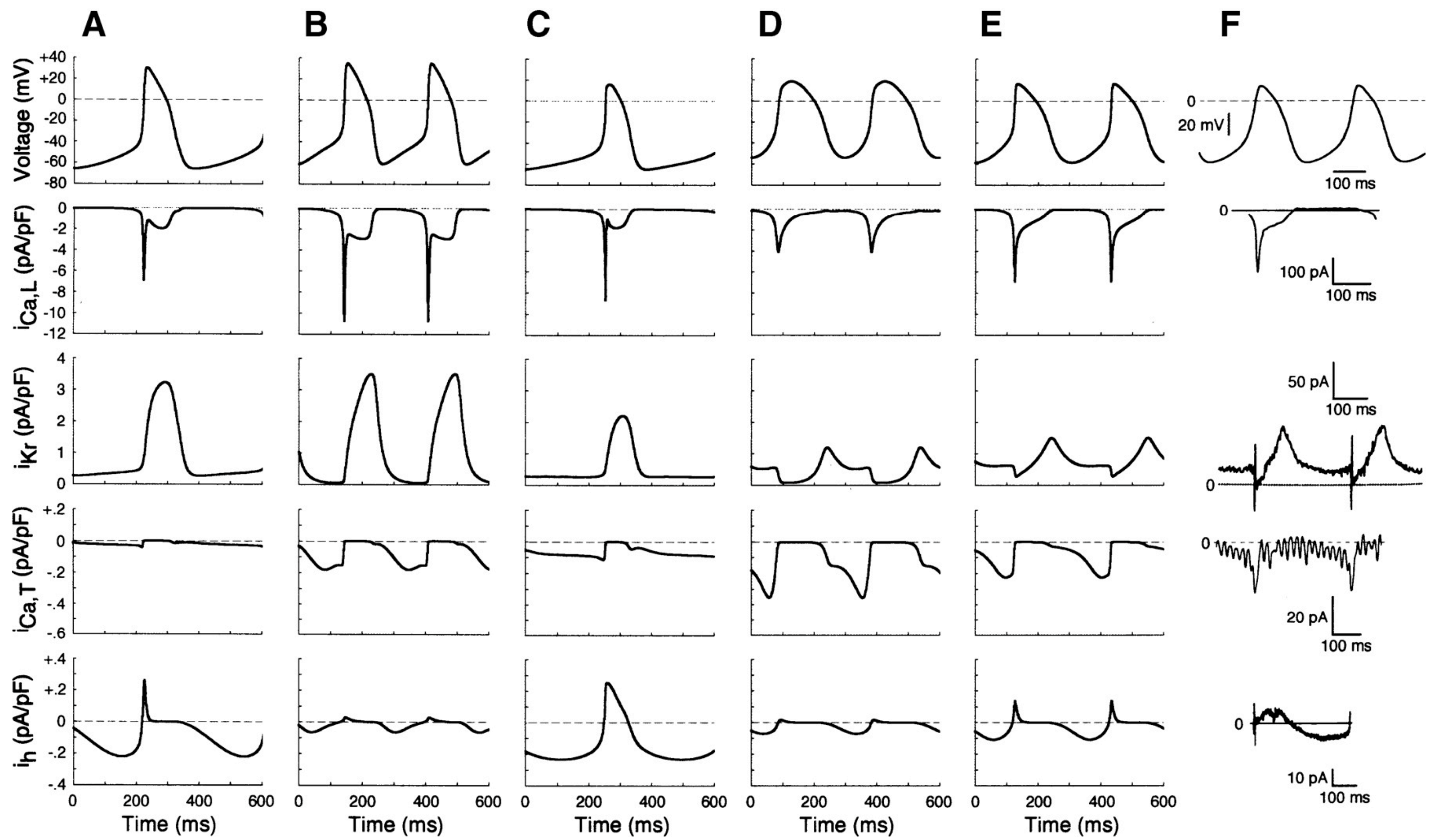
Cardiac ionic model surge

- Surge in development of cell models
- 66 in total (at CellML)
- Different species, regions, pathologies
- Multiple models for the same species/ region/condition





Five different rabbit SAN models

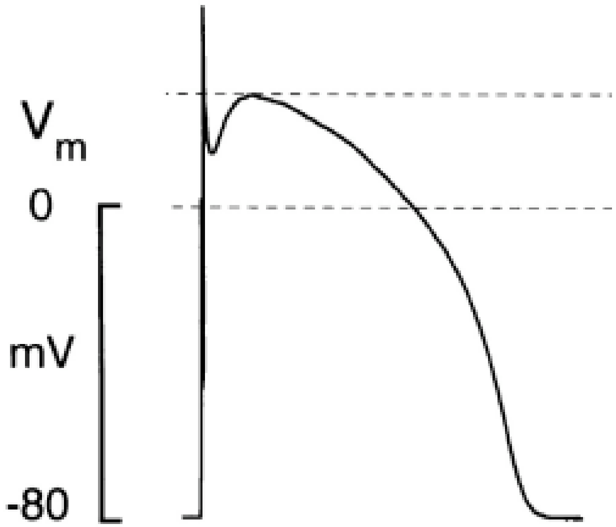


Different models, different action potential shapes and duration

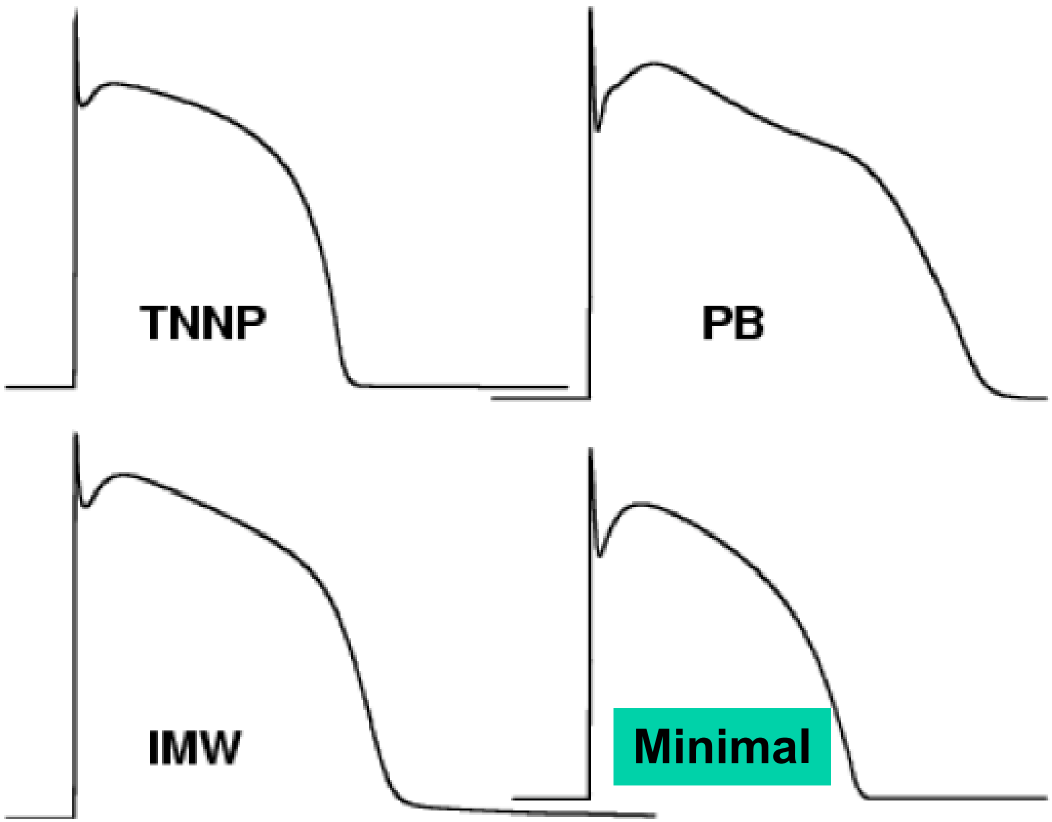
Kurata et al. (2002) AJP 283, H2074-2101.

Four different human ventricular cell models

AP shapes are different (qualitatively and quantitatively).



Experimental epicardial AP.
(M. Näbauer *et al.*, Circulation 1996).



Simulated epicardial APs for the different ionic models.

Cherry, KITP seminar 2006

Why do different models of the same species and regions disagree?

- Some models are simply better than others:
  - Uses better data
  - Uses more data from particular species/region
- The models are equally good/bad:
  - Differences reflect electrophysiological heterogeneity
  - Differences reflect different age, sex, etc.

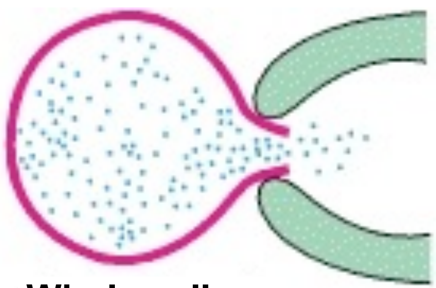
Other modeling considerations

- Models are validated for specific conditions. They may not be valid for your numerical experiments (fast rates, temperature, concentrations, drugs, age, sex).
- A model can give a “right” result for the wrong reason.
- The more complicated the model (more variables and parameters), the more realistically it may behave. However,
- the more complicated the model, the harder it is to pin-point cause-and-effect relationships and the more components may be wrong.
- *Math instead of mice vs. insights from math/physics*

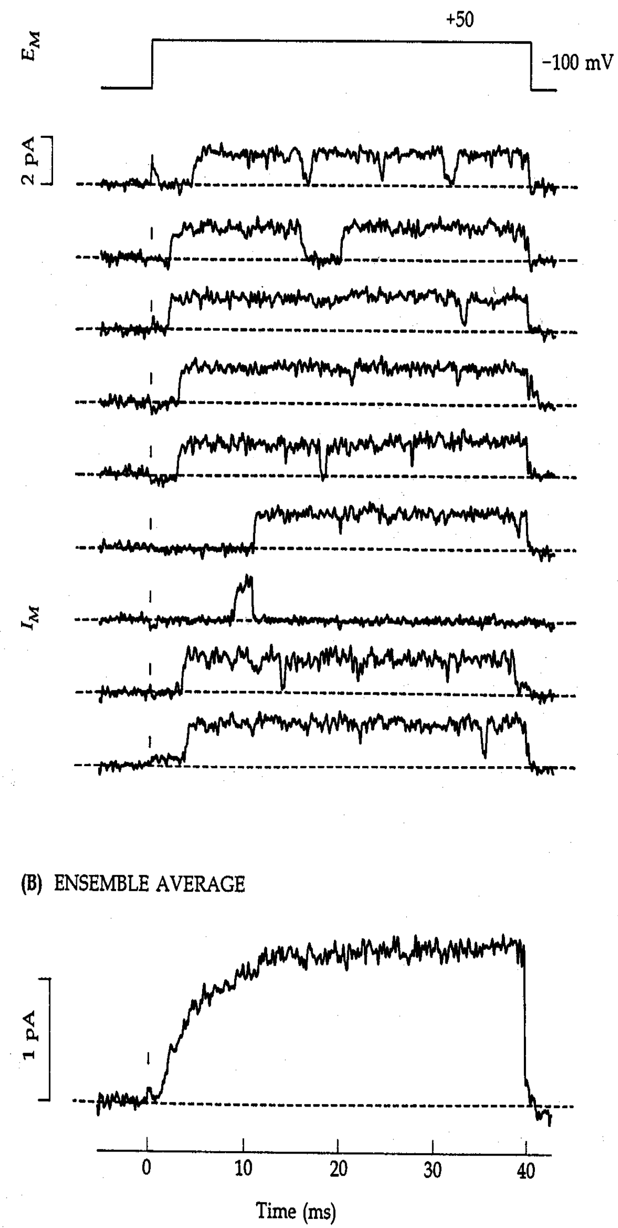
$$\frac{d\mathbf{x}}{dt} = f(\mathbf{x}, t)$$

Multiscale modeling example: single-channel noise

Excised patch



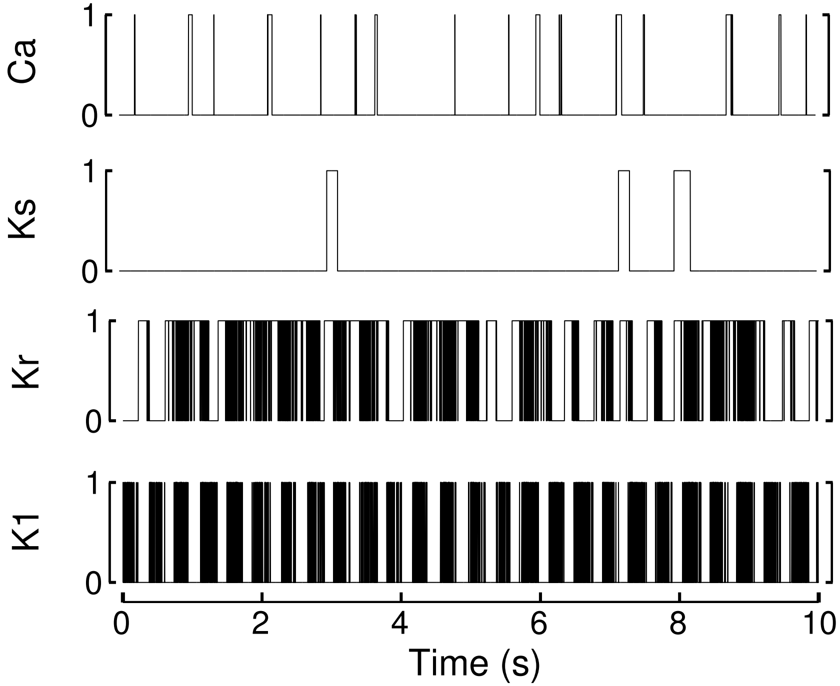
Whole cell

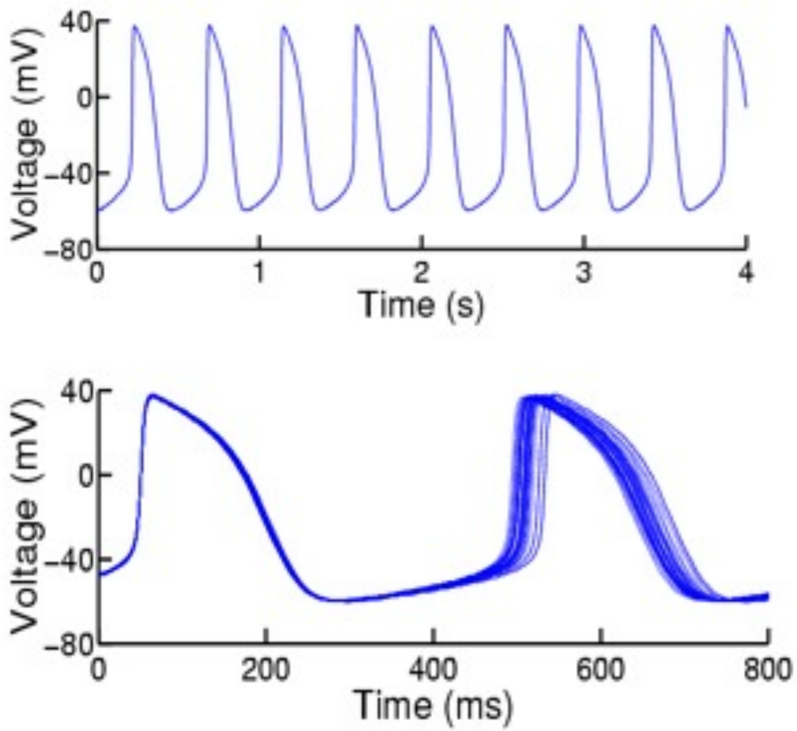


Unitary events add up to give the macroscopic current.

Multiscale modeling example: single-channel noise

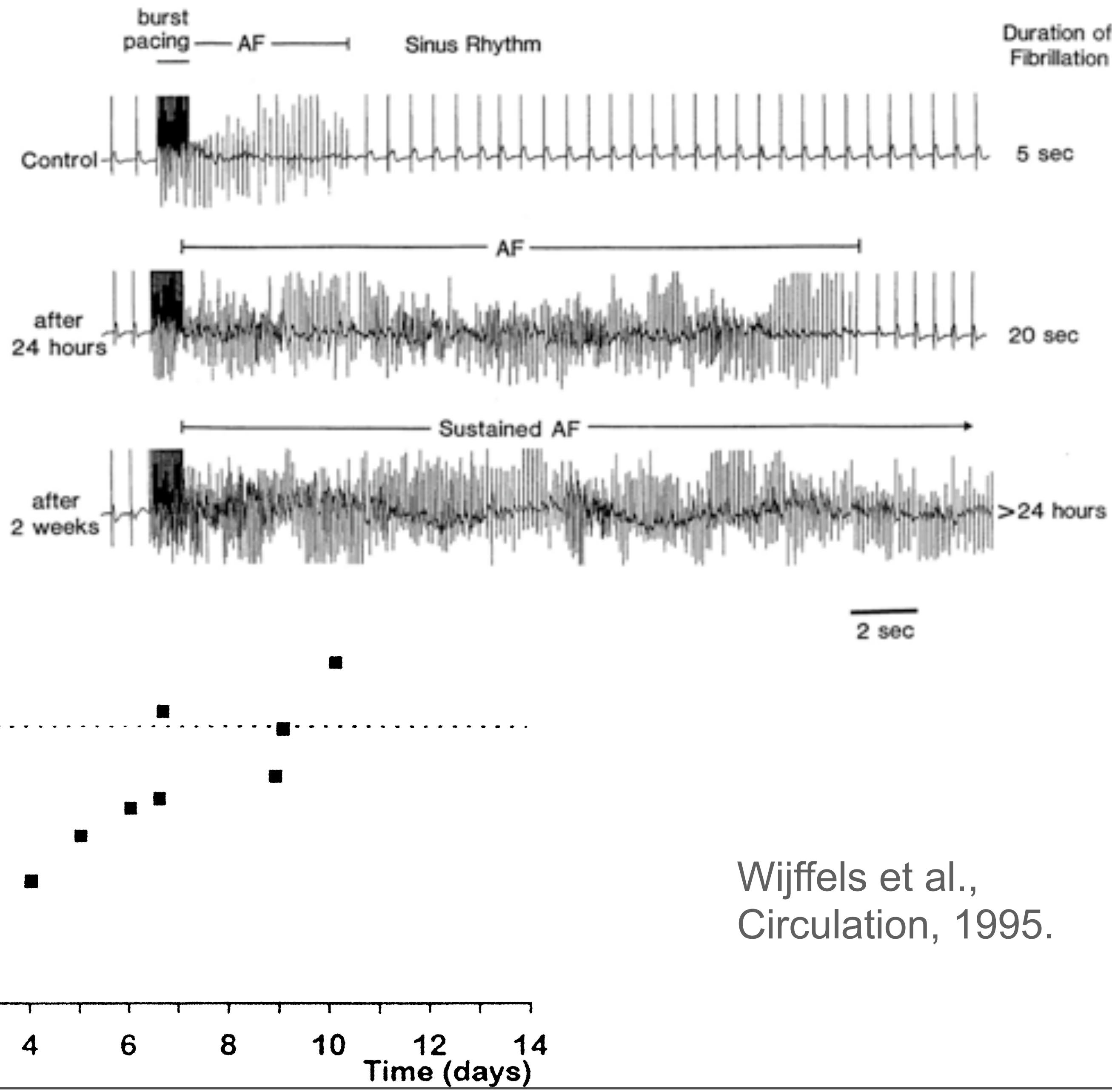
Single channel noise → irregularity of beating





Multiscale modeling example: atrial fibrillation maintenance

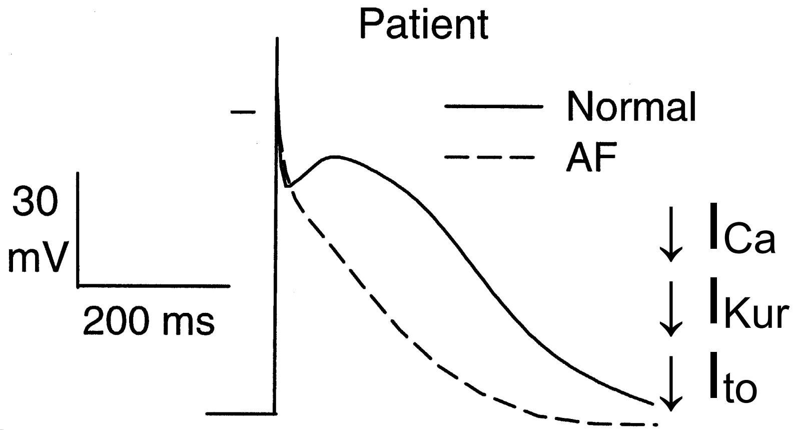
"AF begets AF"



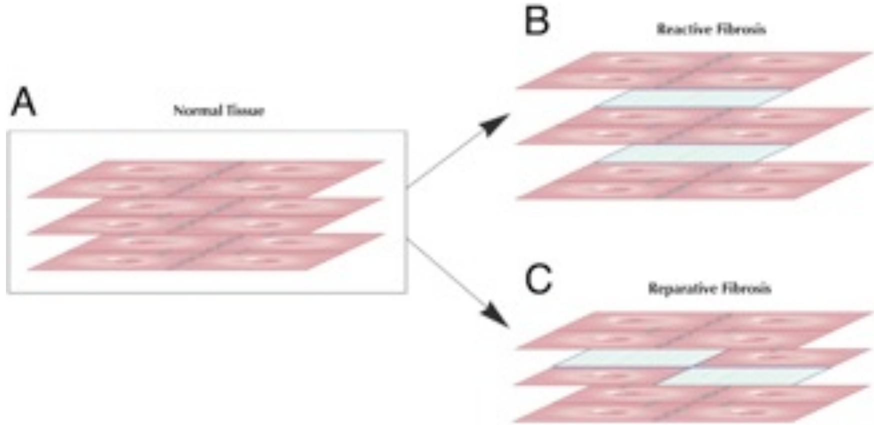
Wijffels et al., Circulation, 1995.

Multiscale modeling example: AF maintenance

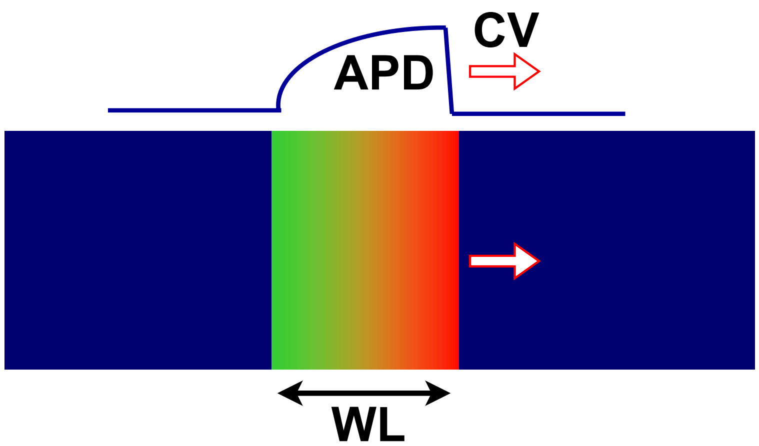
Ionic and structural remodeling



Courtemanche et al.,
Cardiovascular Research, 1999.



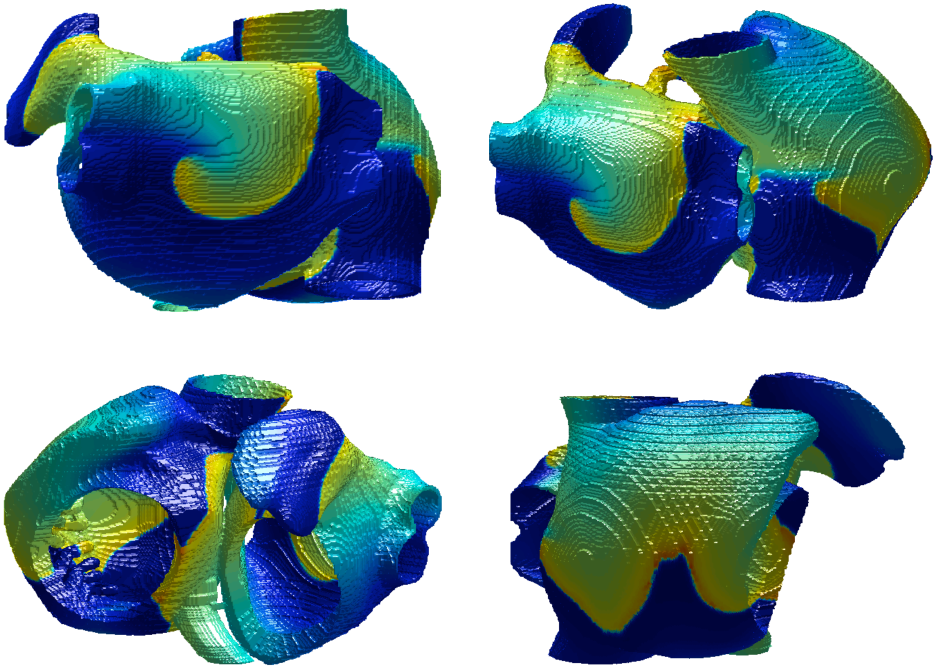
Burstein & Nattel,
J. American College of Cardiology, 2005.



Wave length: WL = CV · APD

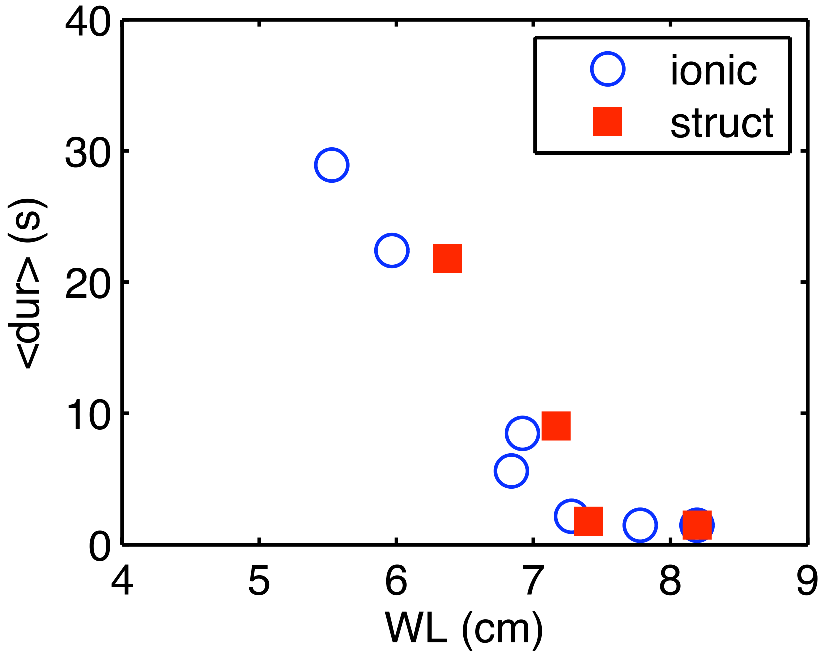
↓ CV
↓ APD ⇨ **↓ WL** ⇨ **multiple waves can fit in the atria**

Multiscale modeling example: AF maintenance



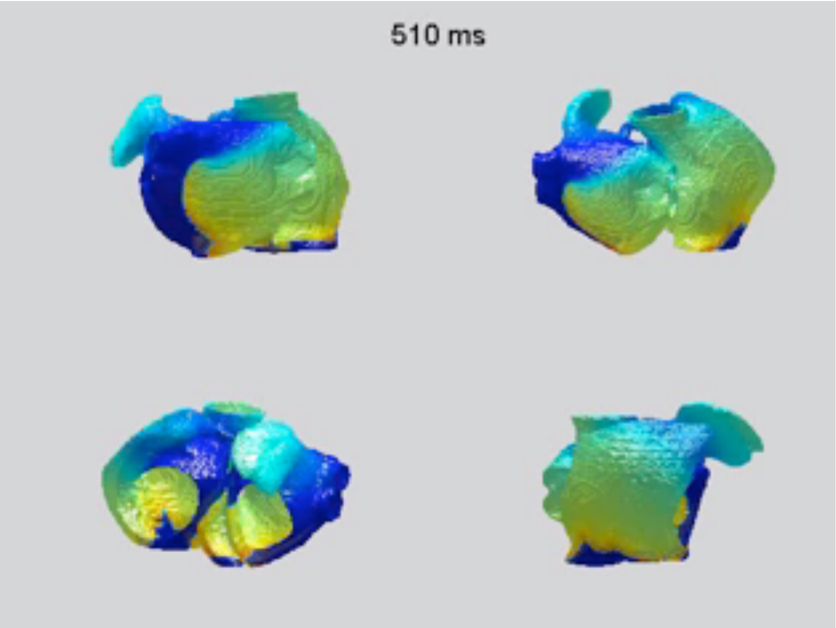
Anatomical structure:
~2,000,000 virtual cells.

Computationally demanding,
but embarrassingly parallel.

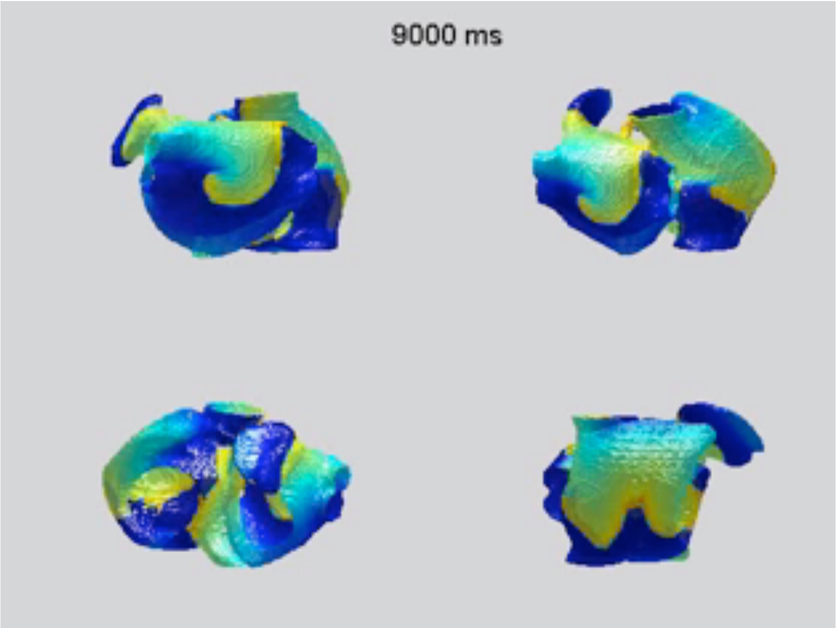


Multiscale modeling example: AF maintenance

No remodeling

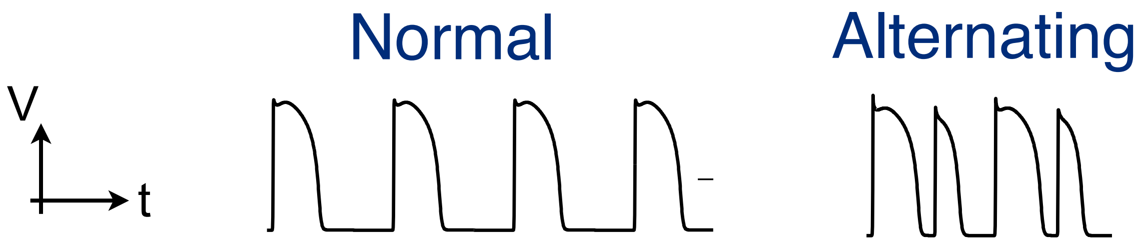


Ionic and structural remodeling

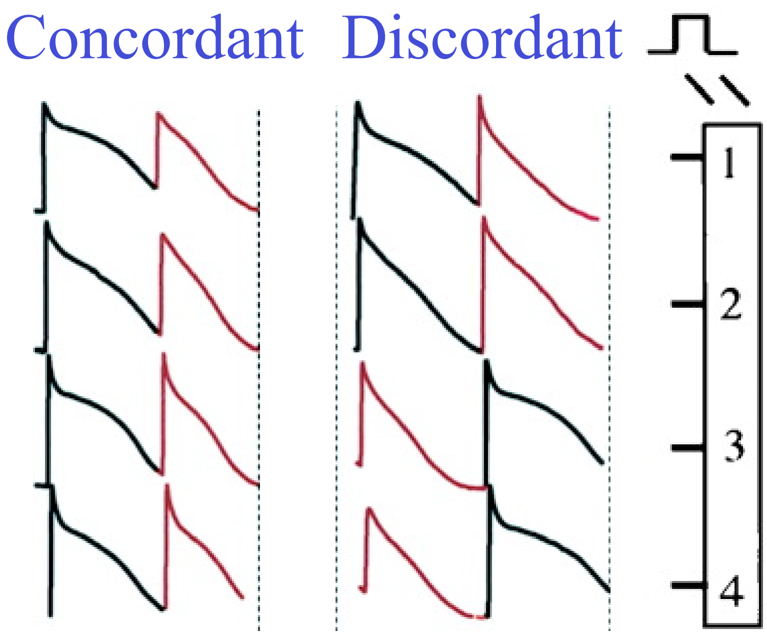


Alternans and its control

Repolarization alternans: a beat-to-beat alternation in action potential duration



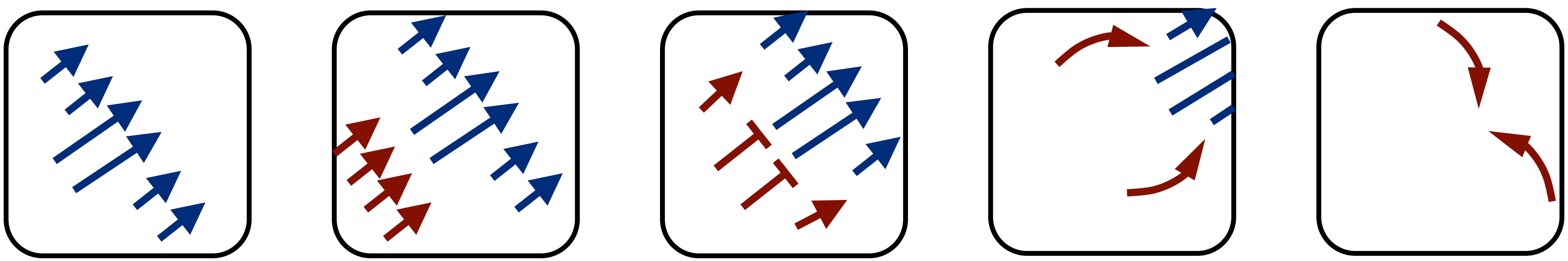
Alternans and arrhythmogenesis



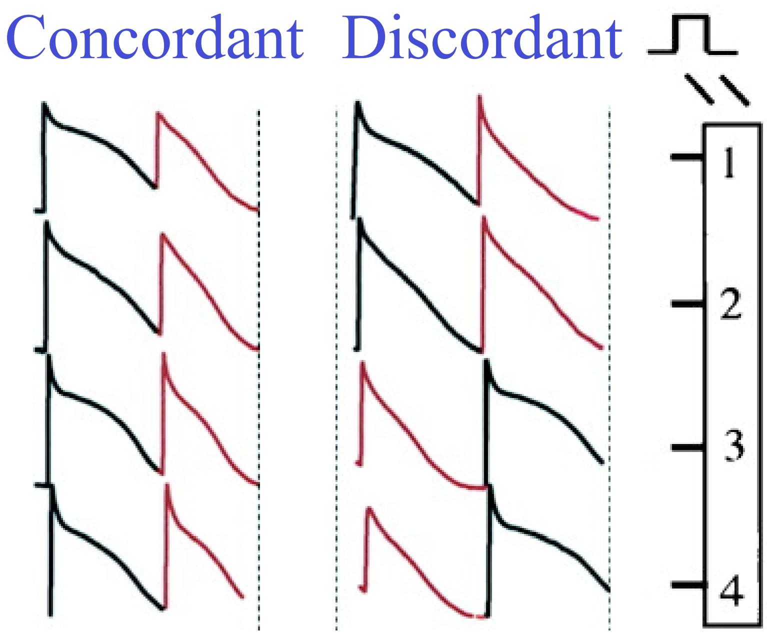
Fox et al. Circ Res 2002

Alternans can induce large repolarization gradients across the heart, ultimately causing unidirectional block.

This may cause life-threatening ventricular tachyarrhythmias.



Alternans and arrhythmogenesis



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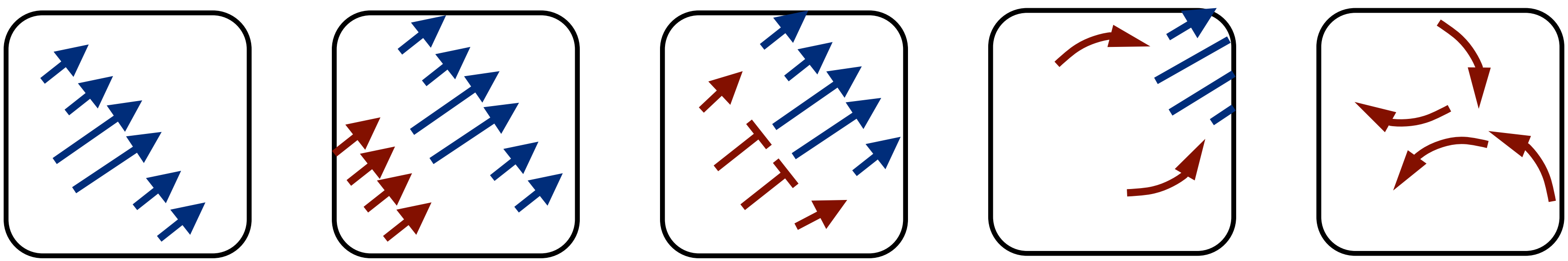

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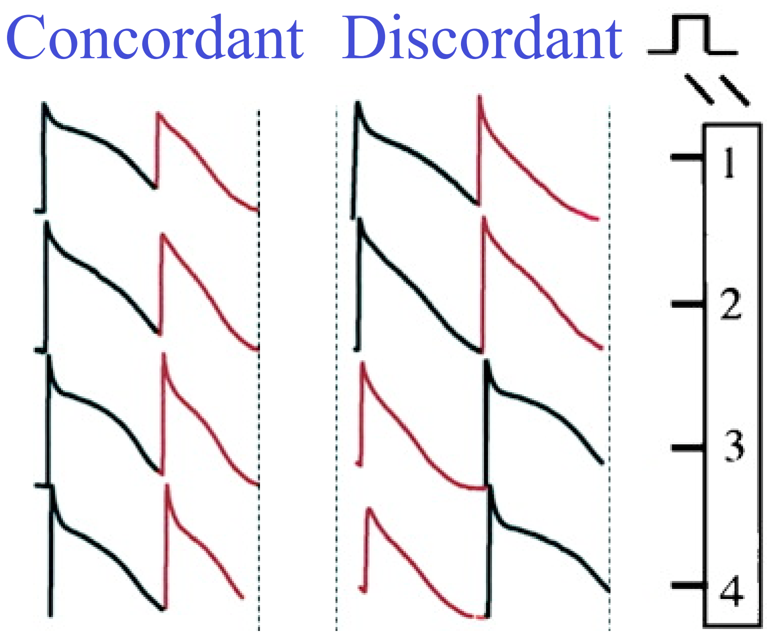

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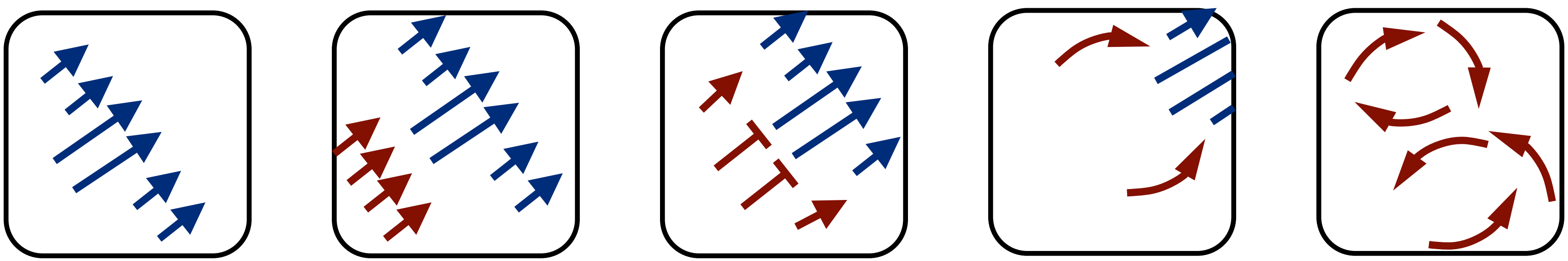


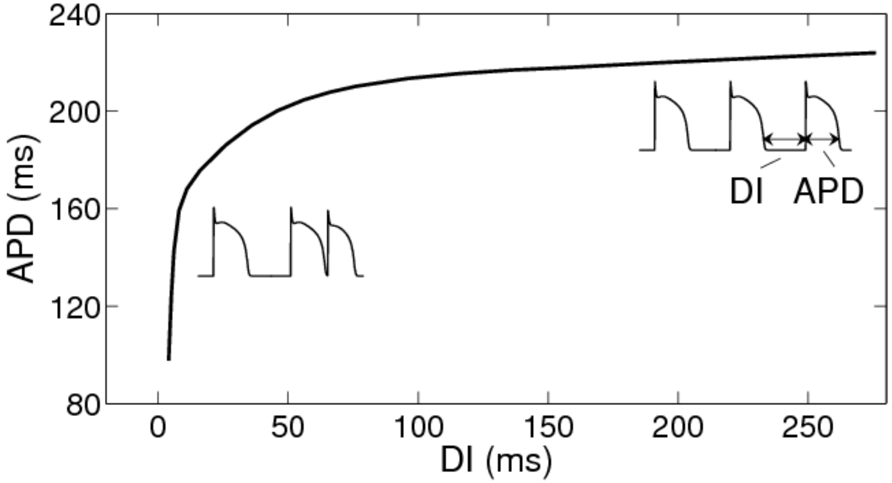
Alternans control

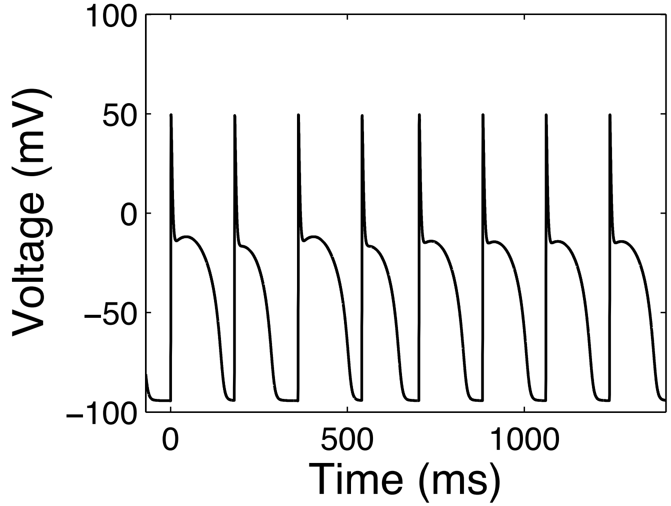
Basic concept: eliminate alternans by applying (small) electrical stimuli at well-timed intervals

$$\mathrm{BCL}_{n+1} = \begin{cases} \mathrm{BCL}^\star & \text{for } \Delta\mathrm{BCL}_{n+1} > 0, \\ \mathrm{BCL}^\star + \Delta\mathrm{BCL}_{n+1} & \text{for } \Delta\mathrm{BCL}_{n+1} \leq 0, \end{cases}$$

with

$$\Delta\mathrm{BCL}_{n+1} = \frac{\gamma}{2}(\mathrm{APD}_{n+1} - \mathrm{APD}_n),$$





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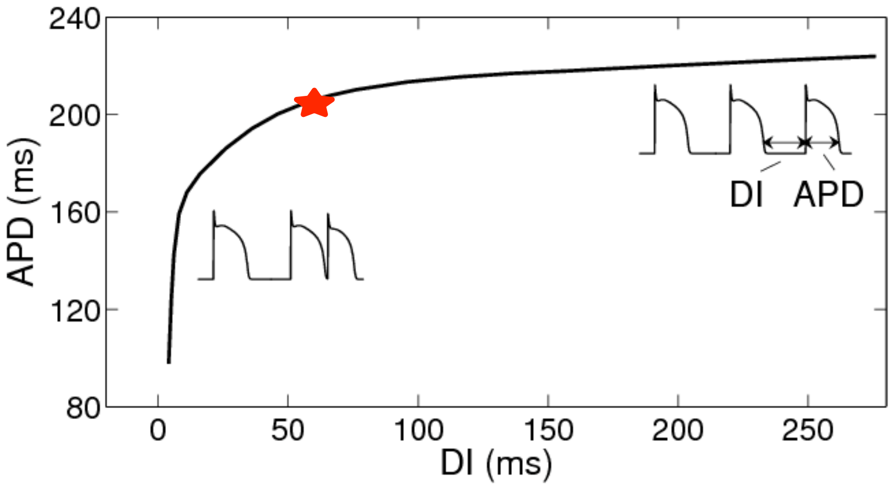

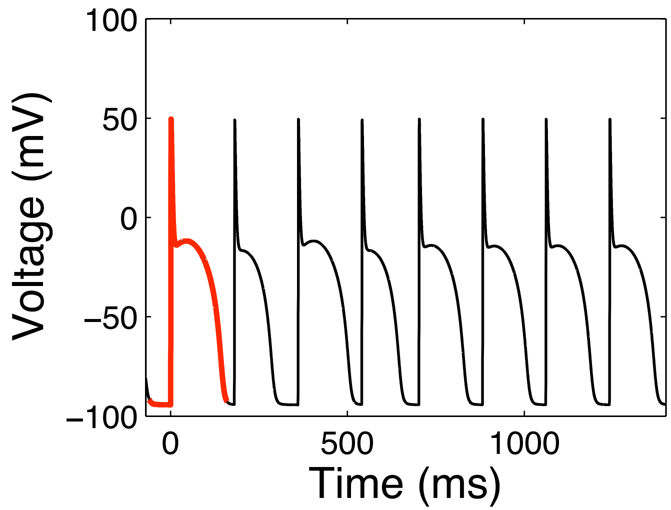

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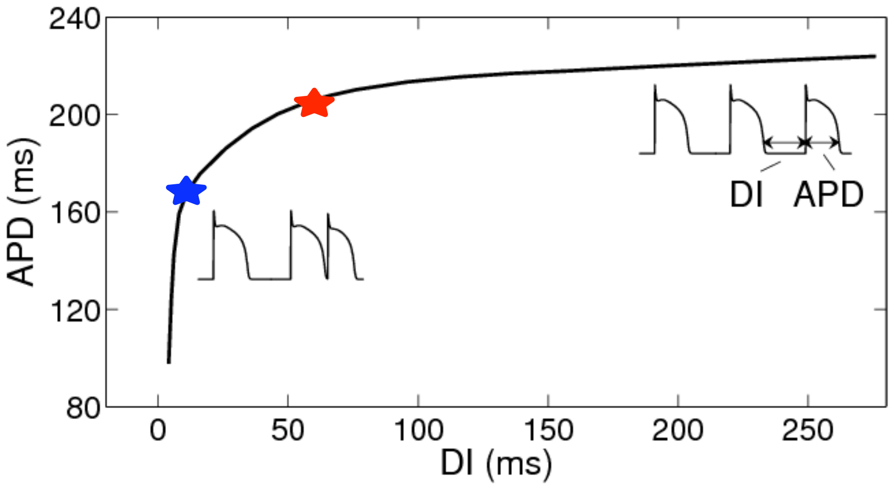

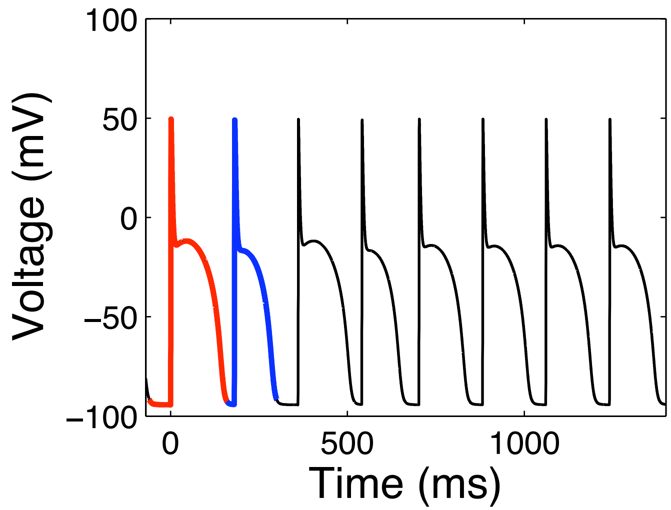

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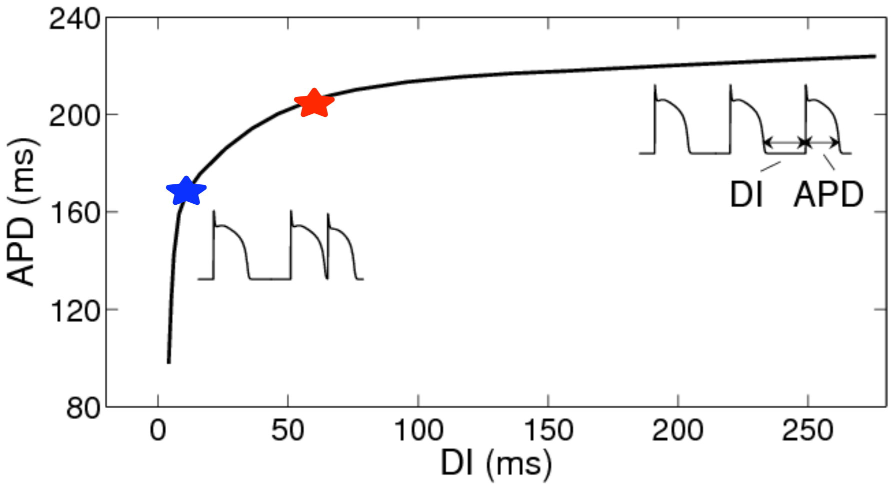

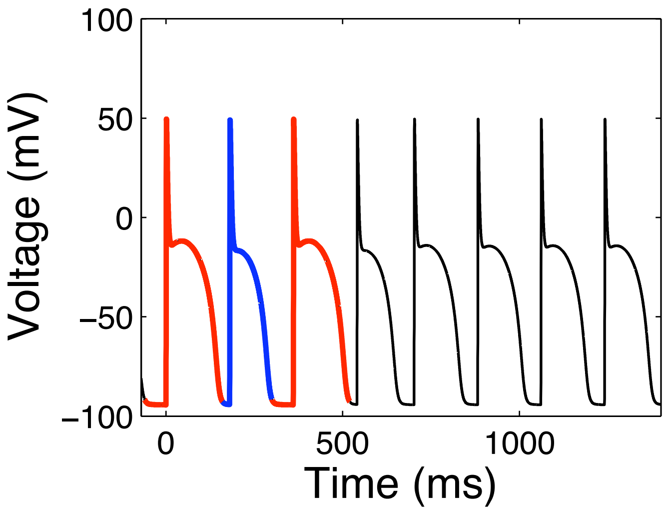

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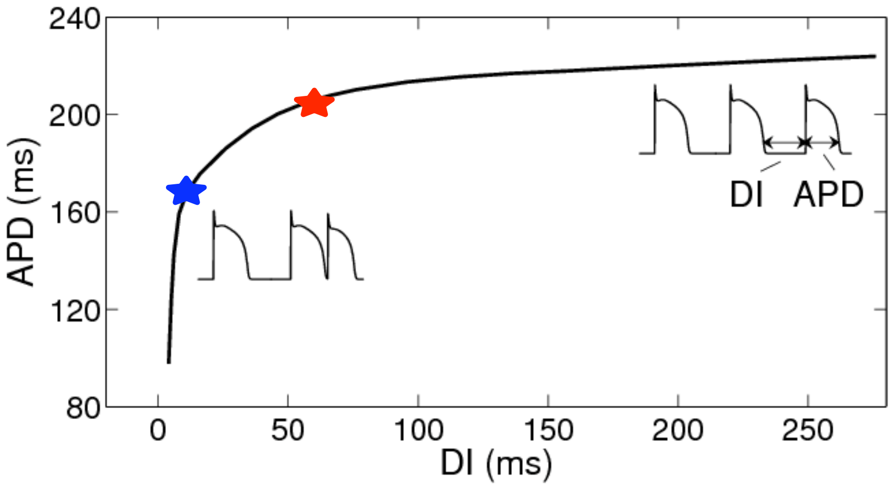

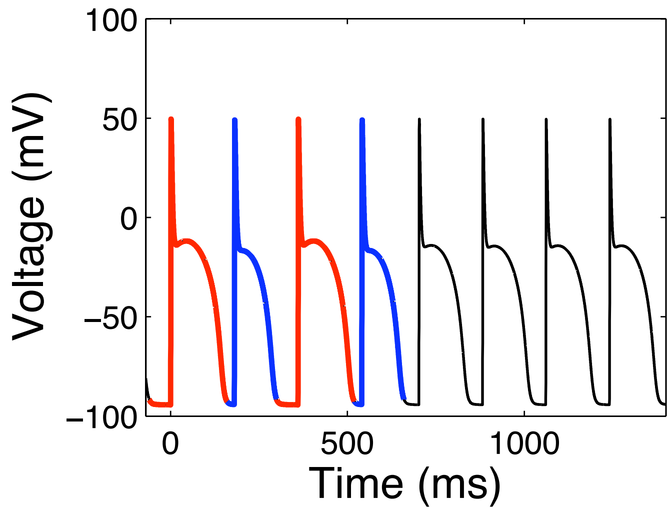

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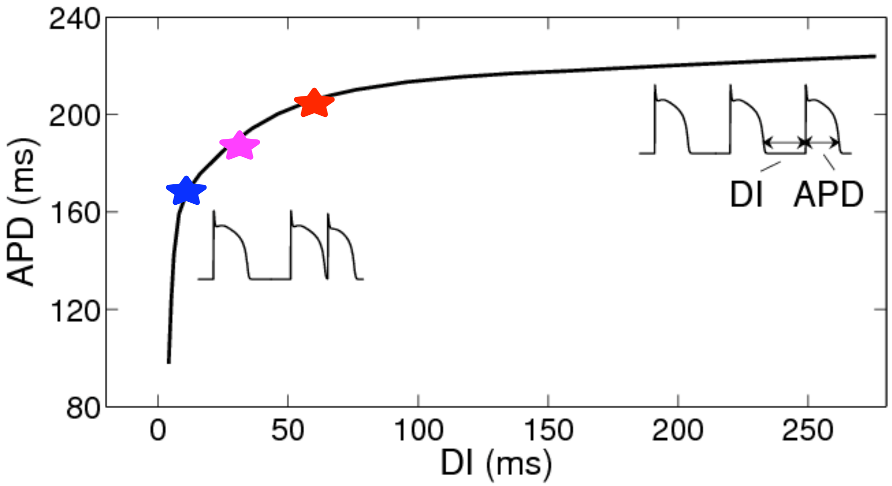
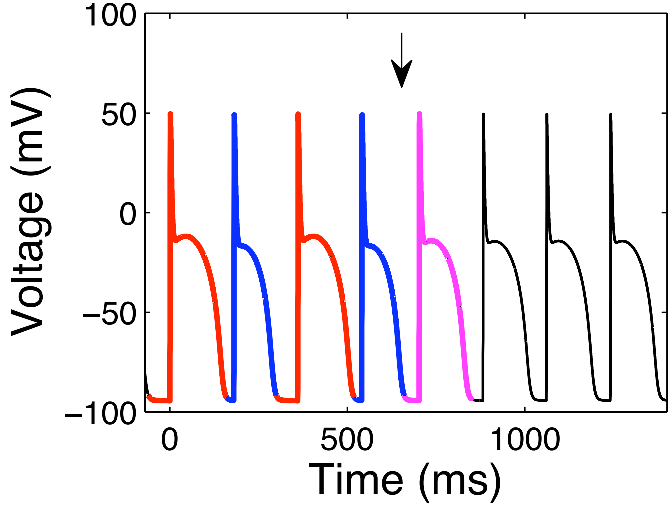
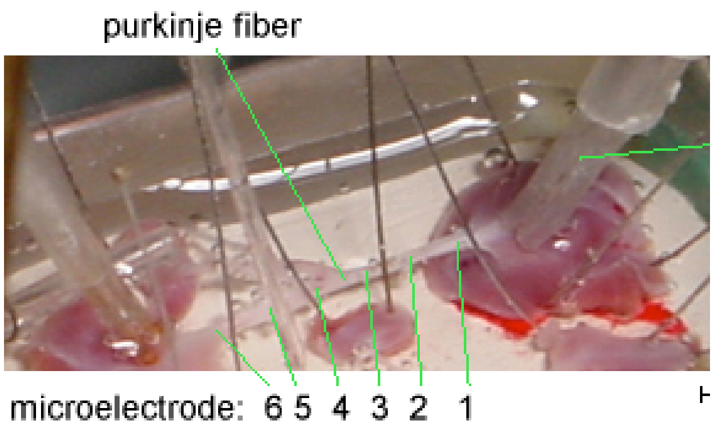
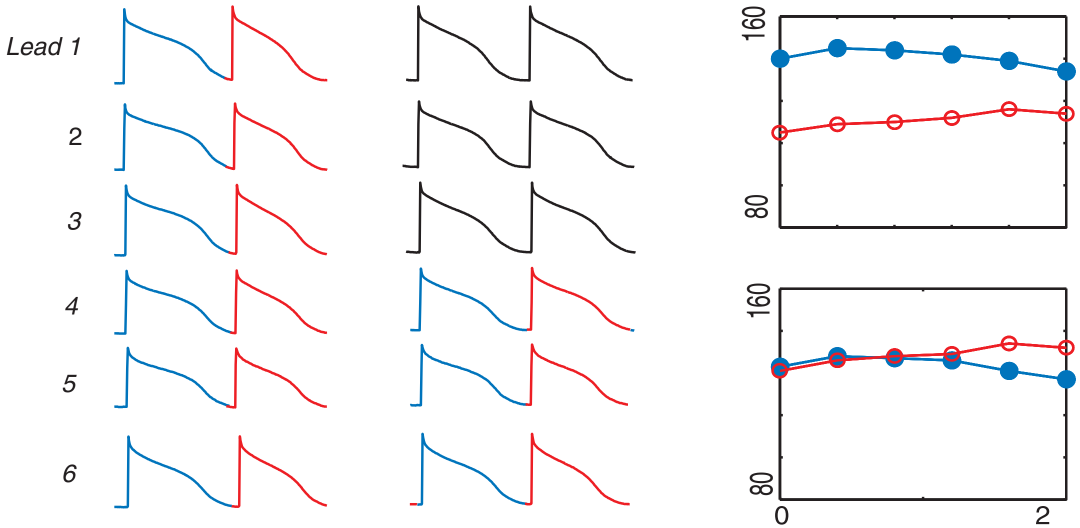


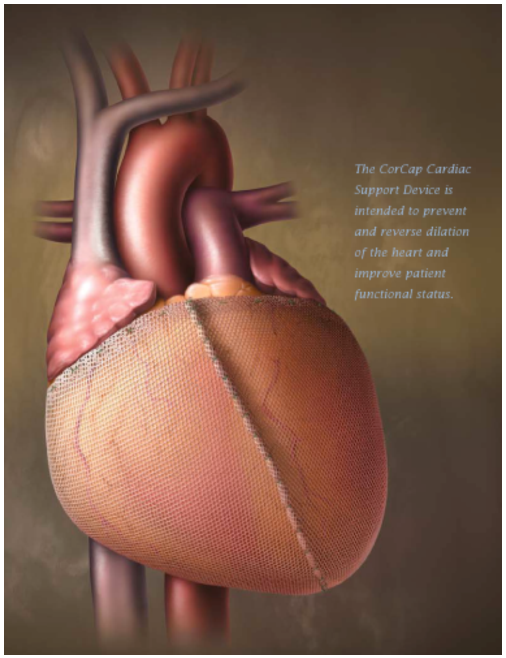


Alternans control works well is single cells but is only effective over ~1 cm in tissue.



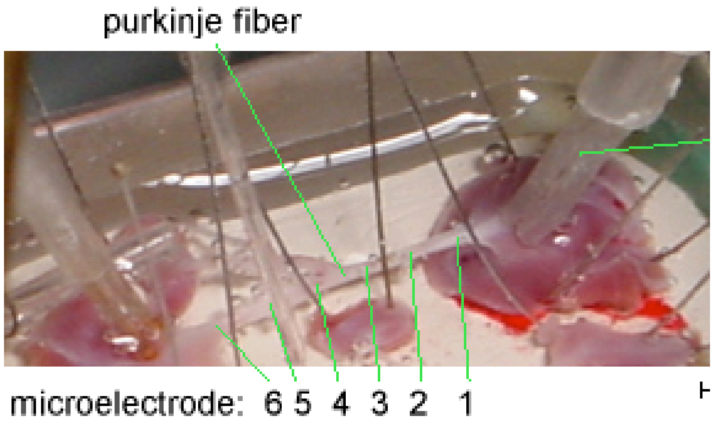


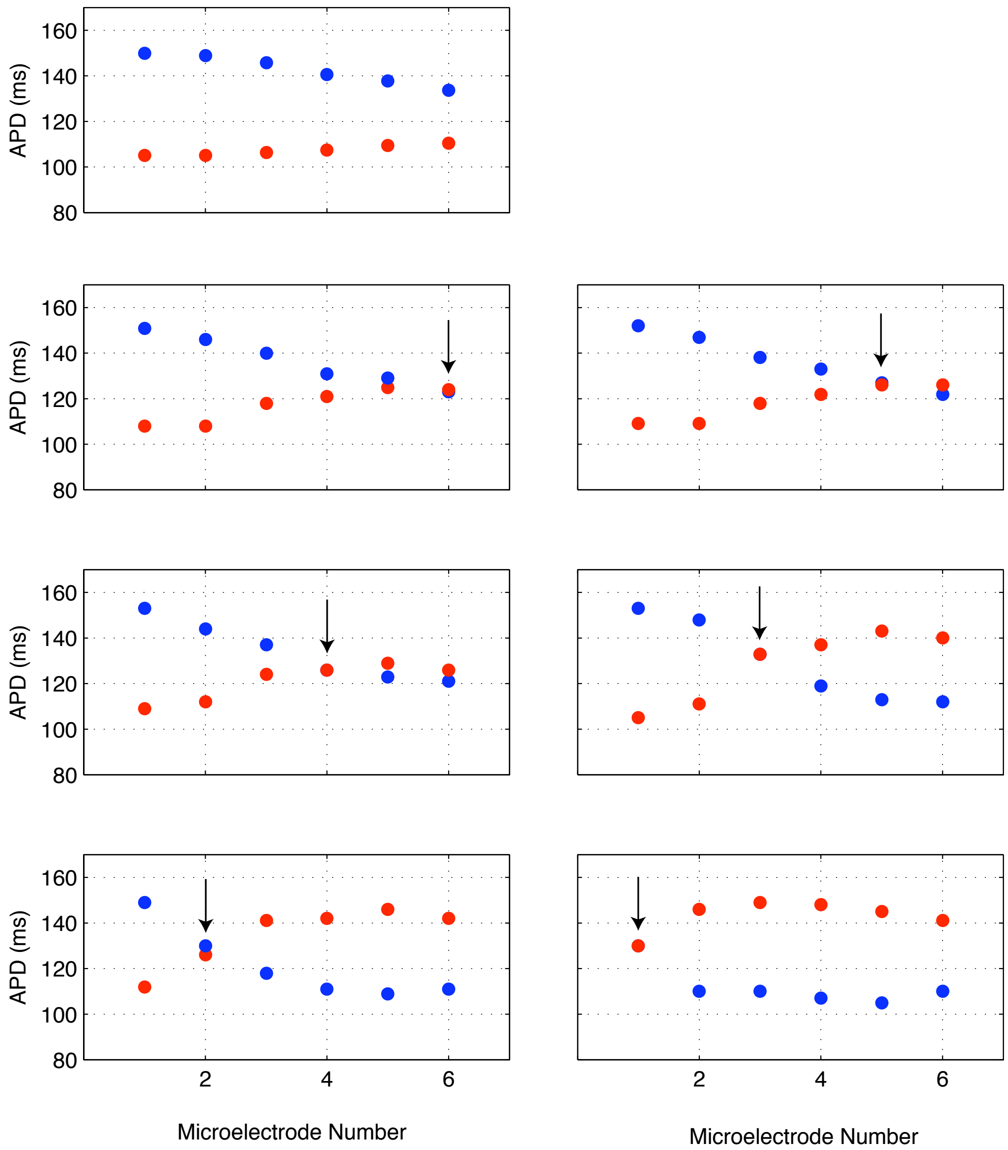
Christini et al., Physical Review Letters, 2006



CorCap Cardiac Support Device: prevent and reverse dilation; add electrode grid?

Off-site alternans control

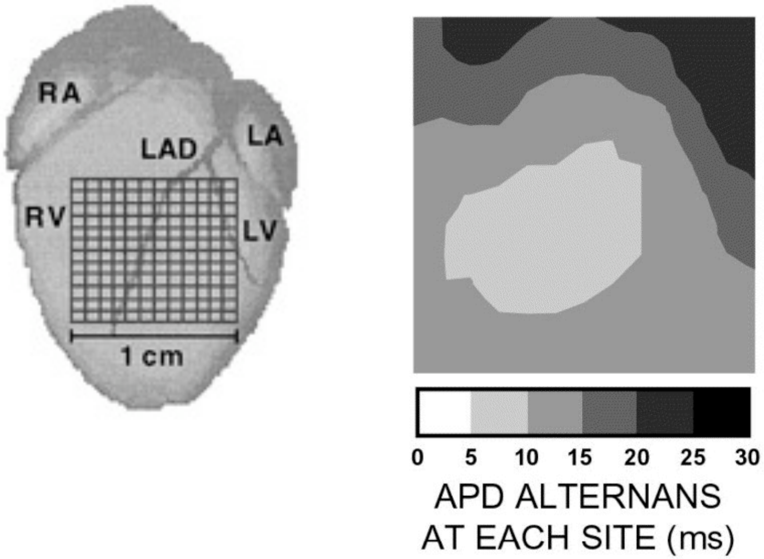




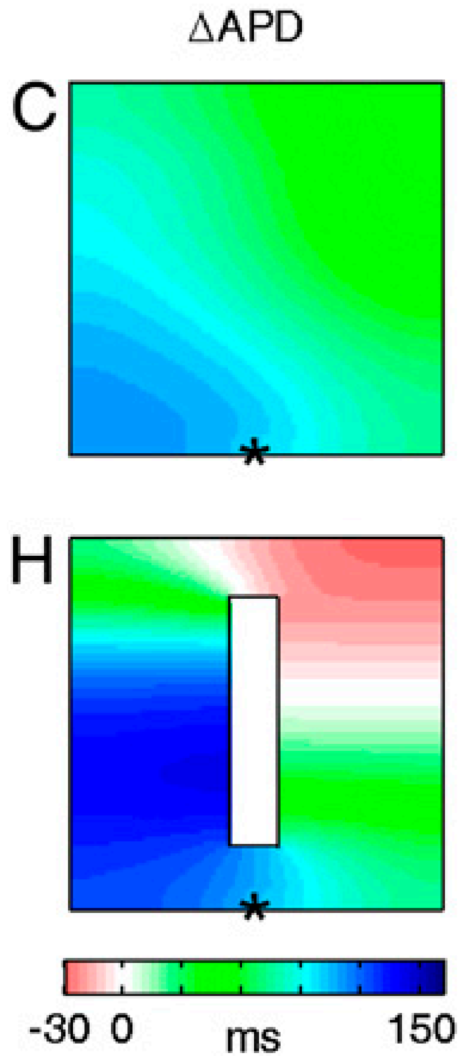
Use data from remote site to control alternans there

Krogh-Madsen et al., Physical Review E, 2010

Use off-site control to eliminate alternans where it's amplitude is large?



Pastore et al.,
Heart Rhythm Journal, 2006



Krogh-Madsen & Christini,
Biophysical Journal, 2007

Summary

- The cardiac action potential is generated by diffusion of ions through specific ion channels in the cell membrane
- Voltage-gated channel dynamics may be described quantitatively by HH-type equations or by Markov models
- Computational models can be used to explain mechanisms of experimentally or clinically observed phenomena