Bayesian Methods
Quantitative Understanding in Biology

Lecture Notes by Jason Banfelder.

Slide Compilation and Demonstratives by David Ballesteros Gomez
Based on slides by Noemi Linden
Let’s say you receive a letter...

Hogwarts School of
Witchcraft and Wizardry

Dear Mrs Naomi Linden,

We are pleased to inform you that you have been accepted at the Hogwarts School of Science and Medicine. Students will be required to report to the Chamber of Reception upon arrival. Term begins on February 30th, 2023. We await your digital owl by no later than January 31st. To avoid detection by Muggles, please use the entrance located in Grand Central Station, Midtown New York, on platform 31415. We look forward to having you in our school.

Yours Sincerely,

Deputy Headmistress

To which house do I belong?
Let’s say you receive a letter...

Hogwarts School of Witchcraft and Wizardry

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Yours Sincerely,

Deputy Headmistress

To which house do I belong?

Does this mean I am a wizard?

Dumbledore would never make a mistake! His mistake rate is 0.01%

There is a 99.99% chance you are a wizard
Let’s say you receive a letter...

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Does this mean I am a wizard?

Dumbledore would never make a mistake! His mistake rate is 0.01%

There is a 99.99% chance you are a wizard.

You are ignoring the fact that magic does not exist, you are a simple human. Don’t forget your priors! Don’t unnecessarily dismiss the information you have.
Bayesian Inference

Bayesian statistics allows you to introduce priors to your probability estimations

$$P(\text{Being a Wizard} \mid \text{Magic does not exist}) \rightarrow 0$$

There is a 99.99% chance you are a wizard

No Harry, you are not a wizard! You probably are a Ph.D. student and if you thought otherwise, you should get some sleep.
Frequentist Statistics

Uses collected data only to draw inferences, there are no assumptions – It is objective.
Frequentist Statistics

Uses collected data only to draw inferences, there are no assumptions – It is objective.

Bayesian Inference

Descries the probability of an event occurrence based on previous knowledge or beliefs of the conditions associated with this event – It is “subjective”.
Frequentist Statistics

Uses collected data only to draw inferences, there are no assumptions – It is objective.

Bayesian Inference

Describes the probability of an event occurrence based on previous knowledge or beliefs of the conditions associated with this event – It is “subjective”.

Relies on the prior model being correct to a certain extent.

Frequentist Probability

Conditional Probability
**Frequentist Statistics**

Uses collected data only to draw inferences, there are no assumptions – It is objective.

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**Bayesian Inference**

Describes the probability of an event occurrence based on previous knowledge or beliefs of the conditions associated with this event – It is “subjective”.

Relies on the prior model being correct to certain extent.

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**Frequentist Probability**

**Conditional Probability**

The number of published medical articles using Bayesian statistics in the period from 1995 to 2018 (Hackenberger, 2019).
Conditional Probability

Height's distribution
Conditional Probability

Solely frequentist approach: Mean; SD; Standard Error $= \frac{SD}{\sqrt{n}}$; CI(95%) = mean ± (1.96 × SEM)
Conditional Probability

What is the probability of selecting a tall person?

**Tall** -> \( T \geq 1.7 \text{ m} \)

\( P(T) = \frac{6}{12} \)
What is the probability of selecting a tall person?

Tall \rightarrow T \geq 1.7\ m

P(T) = \frac{6}{12}

What is the probability of selecting a tall woman?
Conditional Probability

What is the probability of selecting a tall person?
**Tall** -> **T ≥ 1.7 m**
P(T) = 6/12

What is the probability of selecting a **tall woman**?
Conditional Probability

What is the probability of selecting a tall person?

**Tall -> T ≥ 1.7 m**

\[ P(T) = \frac{6}{12} \]

What is the probability of selecting a **tall woman**?

\[ P(\text{Tall}, \text{Female}) = P(\text{Female}) \times P(\text{Tall}) \]

Is this right?

\[ P(\text{Tall}, \text{Female}) = \frac{7}{12} \times \frac{1}{2} = 0.29 \]
Conditional Probability

What is the probability of selecting a tall person?
**Tall -> T ≥ 1.7 m**
P(T) = 6/12

What is the probability of selecting a tall woman?

\[
P(\text{Tall, Female}) = \frac{7}{12} \times \frac{1}{2} = 0.29
\]

We know that these are not independent variables.

Is this right? No
Conditional Probability

What is the probability of selecting a tall person?
Tall $\rightarrow$ $T \geq 1.7 \text{ m}$
P(T) = 6/12

What is the probability of selecting a tall woman?

Joint probability $\rightarrow$ $P(\text{Tall, Female}) = P(\text{Female}) \times P(\text{Tall | Female})$

We know that these are not independent variables.

Conditional probability
What is the probability that a person is Tall given that she is a Female?

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What is the probability that a person is Tall given that she is a Female?
What is the probability that a person is Tall given that she is a Female?

Conditional probability \( \Rightarrow \) \( P(\text{Tall} \mid \text{Female}) \)

\( P(T \mid F) = 2/7 \)  

How many tall women there are

\( T \geq 1.7 \text{ m} \)

\( P(T) = 1/2 \)

\( P(T \mid F) = 2/7 \)
What is the probability that a person is **Tall** given that she is a **Female**?

**Conditional probability** → $P(\text{Tall} \mid \text{Female})$

$P(\text{Tall} \mid \text{Female}) = \frac{2}{7}$

How many tall women there are

$P(\text{Tall}) = \frac{1}{2}$

$P(\text{Tall} \mid \text{Female}) = \frac{2}{7}$

$P(\text{Female}) = \frac{7}{12}$

Joint probability → $P(\text{Tall}, \text{Female}) = P(\text{Female}) \times P(\text{Tall} \mid \text{Female})$
What is the probability that a person is Tall given that she is a Female?

Conditional probability \( \Rightarrow \) \( P(\text{Tall} \mid \text{Female}) \)

\[
P(T \mid F) = \frac{2}{7}
\]

How many tall women there are

Joint probability \( \Rightarrow \) \( P(\text{Tall} , \text{Female} ) = P(\text{Female} ) \times P(\text{Tall} \mid \text{Female} ) \)

\[
P(\text{Tall} , \text{Female} ) = \frac{7}{12} \times \frac{2}{7} = \frac{1}{6}
\]
What is the probability of selecting a tall woman? \( \frac{1}{6} \) ✓

What is the probability of selecting a woman that is tall?

Height's distribution
Conditional Probability

What is the probability of selecting a **tall woman**? \( \frac{1}{6} \) \( \checkmark \)

What is the probability of selecting a **woman** that is **tall**?

Joint probability \( \Rightarrow \) \( P(\text{Female} \ , \ \text{Tall}) = P(\text{Tall}) \times P(\text{Female} \mid \text{Tall}) \)
What is the probability that a person is a Woman given that they are Tall?

\[ P(\text{Female} \cap \text{Tall}) = P(\text{Tall}) \cdot P(\text{Female} | \text{Tall}) \]

What is the probability of selecting a tall woman? \( \frac{1}{6} \) \( \checkmark \)

What is the probability of selecting a woman that is tall?

Joint probability \( \Rightarrow \) \( P(\text{Female}, \text{Tall}) = P(\text{Tall}) \cdot P(\text{Female} | \text{Tall}) \)

What is the probability that a person is a Woman given that they are Tall?
What is the probability of selecting a **tall** woman? \[ \frac{1}{6} \; \checkmark \]

**What is the probability of selecting a woman that is tall?**

Joint probability \[ \Rightarrow \; P(\text{Female}, \text{Tall}) = P(\text{Tall}) \times P(\text{Female} \mid \text{Tall}) \]

\[ \downarrow \]

**What is the probability that a person is a **Woman** given that they are **Tall**?**

Is the conditional probability the same as before?
What is the probability that a person is a Woman given that they are Tall?

\[ P(\text{Female} \cap \text{Tall}) = P(\text{Tall}) \times P(\text{Female} | \text{Tall}) \]

What is the probability of selecting a woman that is tall?

\[ \frac{1}{6} \checkmark \]

What is the probability of selecting a tall woman?

Joint probability \[ \rightarrow \]

\[ P(\text{Female} \cap \text{Tall}) = P(\text{Tall}) \times P(\text{Female} | \text{Tall}) \]

What is the probability that a person is a Woman given that they are Tall?

Is the conditional probability the same as before? No

Conditional probability \[ \rightarrow \]

\[ P(\text{Female} | \text{Tall}) = \frac{2}{6} \]
Conditional Probability

What is the probability of selecting a **tall woman**? \( \frac{1}{6} \) \( \checkmark \)

**What is the probability of selecting a woman that is tall?**

Joint probability \( \Rightarrow \) \( P(\text{Female, Tall}) = P(\text{Tall}) \times P(\text{Female | Tall}) \)

\( \downarrow \)

What is the probability that a person is a **Woman** given that they are **Tall**?

Is the conditional probability the same as before? No

Conditional probability \( \Rightarrow \) \( P(\text{Female | Tall}) = \frac{2}{6} \)

Joint probability \( \Rightarrow \) \( P(\text{Female, Tall}) = P(\text{Tall}) \times P(\text{Female | Tall}) = \frac{1}{2} \times \frac{2}{6} = \frac{1}{6} \)
Conditional Probability

\[
P(Tall, Female) = P(Female) \times P(Tall | Female) = \frac{7}{12} \times \frac{2}{7} = \frac{1}{6}
\]

\[
P(Female, Tall) = P(Tall) \times P(Female | Tall) = \frac{1}{2} \times \frac{2}{6} = \frac{1}{6}
\]
Bayes' Rule

Given that \( P(F, T) = P(T, F) \)

Then \( P(T) \times P(F \mid T) = P(F) \times P(T \mid F) \)

Thus \( P(F \mid T) = \frac{P(F) \times P(T \mid F)}{P(T)} \)

\( P(A \mid B) = \frac{P(A) \times P(B \mid A)}{P(B)} \)

\( P(A) \times P(B \mid A) \) is the Likelihood

\( P(B) \) is the Marginal Probability

\( P(A \mid B) \) is the Posterior Probability

\( P(A) \) and \( P(B \mid A) \) are the Prior and Likelihood, respectively.
Applying Bayes' Rule

A common and easy example of Bayes’ rule being applied are Clinical Tests

- 1% of women in a given population have breast cancer. Prevalence = 1%

- If a woman has breast cancer, there is a 90% chance that a particular diagnostic test will return a positive result. Sensitivity = 90%

- If a woman does not have breast cancer, there is a 10% chance that this diagnostic test will return a positive result. Specificity = 90%

This looks like a reasonably good test

If the test is positive, what is the chance that the patient actually has the disease?
what’s your guess?
Applying Bayes' Rule

If the test is positive, what is the chance that the patient actually has the disease?

\[
P(\text{Cancer} \mid \text{Positive Test})
\]
\[
P(\text{Positive Test} \mid \text{Cancer})
\]
\[
P(\text{Cancer} \cap \text{Positive Test})
\]
\[
P(\text{Cancer} \cap \text{Positive Test})
\]
If the test is positive, what is the chance that the patient actually has the disease?

\[ P(\text{Cancer} \mid \text{Positive Test}) \]
\[ P(\text{Positive Test} \mid \text{Cancer}) \]
\[ P(\text{Cancer} \cap \text{Positive Test}) \]
\[ P(\text{Cancer} \cap \text{Positive Test}) \]

Applying Bayes' Rule

\[ P(\text{Cancer} \mid \text{Positive Test}) = \frac{P(\text{Cancer}) \times P(\text{Positive Test} \mid \text{Cancer})}{P(\text{Positive Test})} \]
1% of women in a given population have breast cancer.

If a woman has breast cancer, there is a 90% chance that a particular diagnostic test will return a positive result.

If a woman does not have breast cancer, there is a 10% chance that this diagnostic test will return a positive result.

If the test is positive, what is the chance that the patient actually has the disease?

\[
P(\text{Cancer} \mid \text{Positive Test}) = \frac{P(\text{Cancer}) \times P(\text{Positive Test} \mid \text{Cancer})}{P(\text{Positive Test})}
\]
Applying Bayes' Rule

If the test is positive, what is the chance that the patient actually has the disease?

- 1% of women in a given population have breast cancer.
- If a woman has breast cancer, there is a 90% chance that a particular diagnostic test will return a positive result.
- If a woman does not have breast cancer, there is a 10% chance that this diagnostic test will return a positive result.

$$P(\text{Cancer}) = 0.01$$

$$P(\text{Cancer} \mid \text{Positive Test}) = \frac{P(\text{Cancer}) \times P(\text{Positive Test} \mid \text{Cancer})}{P(\text{Positive Test})}$$
Applying Bayes' Rule

- 1% of women in a given population have breast cancer.
- If a woman has breast cancer, there is a 90% chance that a particular diagnostic test will return a positive result.
- If a woman does not have breast cancer, there is a 10% chance that this diagnostic test will return a positive result.

\[
P(\text{Cancer}) = 0.01 \quad P(\text{Positive Test | Cancer}) = 0.9
\]

\[
P(\text{Cancer | Positive Test}) = \frac{P(\text{Cancer}) \times P(\text{Positive Test | Cancer})}{P(\text{Positive Test})}
\]
Applying Bayes' Rule

If the test is positive, what is the chance that the patient actually has the disease?

• 1% of women in a given population have breast cancer.

• If a woman has breast cancer, there is a 90% chance that a particular diagnostic test will return a positive result.

• If a woman does not have breast cancer, there is a 10% chance that this diagnostic test will return a positive result.

\[ P(\text{Cancer}) = 0.01 \]
\[ P(\text{Positive Test} \mid \text{Cancer}) = 0.9 \]

\[ P(\text{Positive Test}) = \]

\[ P(\text{Cancer} \mid \text{Positive Test}) = \frac{P(\text{Cancer}) \times P(\text{Positive Test} \mid \text{Cancer})}{P(\text{Positive Test})} \]
Applying Bayes' Rule

If the test is positive, what is the chance that the patient actually has the disease?

- 1% of women in a given population have breast cancer.
- If a woman has breast cancer, there is a 90% chance that a particular diagnostic test will return a positive result.
- If a woman does not have breast cancer, there is a 10% chance that this diagnostic test will return a positive result.

\[ P(Cancer) = 0.01 \quad \quad P(Positive \ Test \ | \ Cancer) = 0.9 \]

\[ P(Positive \ Test) = \text{True Positive} + \text{False Positive} \]

\[
P(Cancer \ | \ Positive \ Test) = \frac{P(Cancer) \times P(Positive \ Test \ | \ Cancer)}{P(Positive \ Test)}
\]
Applying Bayes' Rule

If the test is positive, what is the chance that the patient actually has the disease?

- 1% of women in a given population have breast cancer.
- If a woman has breast cancer, there is a 90% chance that a particular diagnostic test will return a positive result.
- If a woman does not have breast cancer, there is a 10% chance that this diagnostic test will return a positive result.

\[
P(\text{Positive Test }\) = \text{ True Positive } + \text{ False Positive}
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Applying Bayes' Rule

If the test is positive, what is the chance that the patient actually has the disease?

- 1% of women in a given population have breast cancer.
- If a woman has breast cancer, there is a 90% chance that a particular diagnostic test will return a positive result.
- If a woman does not have breast cancer, there is a 10% chance that this diagnostic test will return a positive result.

\[
P(\text{Positive Test}) = \text{True Positive} + \text{False Positive}
\]

\[
P(\text{Positive Test}) = P(\text{Positive Test} \mid \text{Cancer}) \times P(\text{Cancer}) + P(\text{Positive Test} \mid \text{No Cancer}) \times P(\text{No Cancer})
\]
Applying Bayes' Rule

If the test is positive, what is the chance that the patient actually has the disease?

- 1% of women in a given population have breast cancer.
- If a woman has breast cancer, there is a 90% chance that a particular diagnostic test will return a positive result.
- If a woman does not have breast cancer, there is a 10% chance that this diagnostic test will return a positive result.

\[
P(\text{Positive Test}) = P(\text{Positive Test} | \text{Cancer}) \times P(\text{Cancer}) + P(\text{Positive Test} | \text{Healthy}) \times P(\text{Healthy})
\]

\[
P(\text{Positive Test}) = \text{True Positive} + \text{False Positive}
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Applying Bayes' Rule

If the test is positive, what is the chance that the patient actually has the disease?

1% of women in a given population have breast cancer.

If a woman has breast cancer, there is a 90% chance that a particular diagnostic test will return a positive result.

If a woman does not have breast cancer, there is a 10% chance that this diagnostic test will return a positive result.

\[
P(\text{Positive Test}) = P(\text{Positive Test} \mid \text{Cancer}) \times P(\text{Cancer}) + P(\text{Positive Test} \mid \text{Healthy}) \times P(\text{Healthy})
\]

\[
P(\text{Positive Test}) = (0.9 \times 0.01) + (0.1 \times (1 - 0.01)) = 0.108
\]
Applying Bayes' Rule

If a woman has breast cancer, there is a 90% chance that a particular diagnostic test will return a positive result.

If a woman does not have breast cancer, there is a 10% chance that this diagnostic test will return a positive result.

1% of women in a given population have breast cancer.

P(Positive Test ) = 0.108

P(Cancer | Positive Test ) = \frac{0.01 \times 0.9}{0.108} = 0.083
Applying Bayes' Rule

A common and easy example of Bayes’ rule being applied are Clinical Tests

- 1% of women in a given population have breast cancer.  \[ \text{Prevalence} = 1\% \]

- If a woman has breast cancer, there is a 90% chance that a particular diagnostic test will return a positive result.  \[ \text{Sensitivity} = 90\% \]

- If a woman does not have breast cancer, there is a 10% chance that this diagnostic test will return a positive result.  \[ \text{Specificity} = 90\% \]

This looks like a reasonably good test  \[ P(\text{Cancer} | \text{Positive Test} ) = \frac{0.01 \times 0.9}{0.108} = 0.083 \]  or not
Health Panel Recommends Screening All Kids 8 and Up for Anxiety

To address the mental health crisis facing American youth, experts also said adolescents should be screened for depression.

Pediatricians should screen children as young as 8 for anxiety and kids 12 and older for depression during routine well checks, the U.S. Preventive Services Task Force said Tuesday.

The recommendation from the independent panel of experts applies to children who aren't showing any signs or symptoms of a mental health problem. Children who are — regardless of age — should be referred for specialized care, task force member Lori Pbert said.
This is not a claim against this recommendation! I don’t know the specifics of this disease in the population. Prevalence, sensitivity and specificity of the screening test and other parameters are important/required to know before any claim.
Bayesian Methods

Bayes theorem can be used to data analysis and model estimation

\[
P(A | B) = \frac{P(A) \times P(B | A)}{P(B)}
\]

Prior Distribution = Background Knowledge

Likelihood Distribution = Observational Data

Posterior Probability = updated knowledge

Marginal Likelihood

(Schoot et al, 2021)
Bayesian Methods

Let’s imagine that we flip a coin...

Flips = 20
Heads = 13

What is the bias of the coin?

```r
binom.test(13, 20)
##
## Exact binomial test
##
## data: 13 and 20
## number of successes = 13, number of trials = 20, p-value = 0.2632
## alternative hypothesis: true probability of success is not equal to 0.5
## 95 percent confidence interval:
## 0.4078115 0.8460908
## sample estimates:
## probability of success
## 0.65
```
Bayesian Methods

Let’s imagine that we flip a coin...

Flips = 20
Heads = 13

What is the bias of the coin?

Frequentist approach

Bias:
There is a **0.65 probability** the coin will land head
This value could also be between 0.41 and 0.85.
Bayesian Methods

Let’s imagine that we flip a coin...

Flips = 20
Heads = 13

What is the bias of the coin?

Frequentist approach

Bayesian approach

Data
Model

Bias:
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Bayesian Methods

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Bayesian approach

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Model?

### Bias:
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Flips = 20
Heads = 13

What is the bias of the coin?

Frequentist approach

Bayesian approach

Data
Model

Bias:
There is a **0.65 probability** the coin will land head
This value could also be between 0.41 and 0.85.
Bayesian Methods

**Observational data** and distribution model

<table>
<thead>
<tr>
<th>Flips</th>
<th>Heads</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>13</td>
</tr>
</tbody>
</table>

What is the probability of getting this outcome with a fair coin?

\[ \text{dbinom}(13, \text{size} = 20, \text{prob} = 0.5) \]

```
## [1] 0.07392883
```

What is the probability of getting this outcome with a biased coin?

\[ \text{dbinom}(13, \text{size} = 20, \text{prob} = 0.25) \]

```
## [1] 0.0001541923
```
Bayesian Methods

Observational data and distribution model

Flips = 20
Heads = 13

What is the most likely bias?

```r
coin.bias <- seq(from = 0, to = 1, by = 0.01)
likelihood <- dbinom(13, 20, prob = coin.bias)
barplot(likelihood, names.arg = coin.bias, ylab = "likelihood")
```

The maximum likelihood is around 0.65
Bayesian Methods

Observational data and distribution model

Collection of 101 coins
\[ P_n \text{ coin} = \frac{1}{101} = 0.0099 \]
Bayesian Methods

Observational data and **distribution model**

Flips = 20
Heads = 13

What is the probability of getting the biased coin(n) given the data that we observed?

\[ P(A | B) = \frac{P(A) \times P(B | A)}{P(B)} \]

\[ P(M_{0.5} | D_{13}) = \frac{P(M_{0.5}) \times P(D_{13} | M_{0.5})}{P(D_{13})} \]

Collection of 101 coins
\[ P_n \text{ coin} = \frac{1}{101} = 0.0099 \]
Bayesian Methods

\[
\frac{1}{101} = 0.0099
\]

\[
P(M_{0.5} \mid D_{13}) = \frac{P(M_{0.5}) \times P(D_{13} \mid M_{0.5})}{P(D_{13})}
\]

```r
dbinom(13, size = 20, prob = 0.5)
#> [1] 0.07392883
```
Bayesian Methods

\[
\frac{1}{101} = 0.0099
\]

\[
P(M_{0.5} \mid D_{13}) = \frac{P(M_{0.5}) \times P(D_{13} \mid M_{0.5})}{P(D_{13})}
\]

\[
P(D_{13}) = P(D_{13} \mid M_{0.00}) \times P(M_{0.00}) + P(D_{13} \mid M_{0.01}) \times P(M_{0.01}) + ... + P(D_{13} \mid M_{1}) \times P(M_{1})
\]

\[
(p.d13 <- \text{sum(dbinom(13, 20, coin.bias) * (1 / 101)))}
\]

\[
## [1] 0.04714757
\]
Bayesian Methods

\[ P(M_{0.5} \mid D_{13}) = \frac{0.009 \times 0.07}{0.047} = 0.015 \]
Bayesian Methods

\[ P(M_x \mid D_{13}) = \frac{0.009 \times P(D_{13} \mid M_x)}{0.047} \]

```r
posterior.probability <- dbinom(13, 20, coin.bias) * (1 / 101) / p.d13
sum(posterior.probability)

## [1] 1

barplot(posterior.probability, names.arg = coin.bias)
```

What would be a more interesting question?
Bayesian Methods

Bayesian methods allow for the exploration of different models and expectations.

\[
P(A \mid B) = \frac{P(A) \times P(B \mid A)}{P(B)}
\]

- **Prior Distribution** $\rightarrow$ Background Belief
- **Likelihood Distribution** $=$ Observational Data
- **Marginal Likelihood**
- **Posterior Probability** $=$ updated knowledge
Prior/Model

All biases are equally likely

Data

Flips = 20
Heads = 13

\[ P(M_x \mid D_{13}) = \frac{P(D_{13} \mid M_x) \times M(n)}{0.07} = \]
Priors in Bayesian Methods

What if there is no good reason to think the coin is biased?

Prior:

- Uniform Distribution
- Bell shape distribution
Priors in Bayesian Methods

Prior/Model

- Fair coin

Data

- Flips = 20
- Heads = 13

\[
P(M_x | D_{13}) = \frac{P(D_{13} | M_x) \times M(n)}{0.05}
\]

(p.d13 <- sum(dbinom(13, 20, coin.bias) * prior.probability))

# [1] 0.05205051
Priors in Bayesian Methods

Prior/Model
Fair coin

Data
Flips = 200
Heads = 130

More Data

Data overwhelms the prior

\[
P(M_x \mid D_{13}) = \frac{P(D_{13} \mid M_x) \times M(x)}{0.004} = \]

\[
\text{p.d130 <- sum(dbinom(130, 200, coin.bias) * prior.probability_2)}
\]

\[
\text{print(p.d130)}
\]

\[
[1] 0.004049461
\]
Priors in Bayesian Methods

Prior/Model

We know it's biased

Data

Flips = 10
Heads = 6

Complex Interaction of priors vs data

\[ P(M_x | D_{13}) = \frac{P(D_{13} | M_x)}{0.1} \times M(x) \]
Priors in Bayesian Methods

Possibilities and consequences

You can explore virtually any prior you want.

Be careful with your prior. Ideally, they should be supported by previous knowledge, even more if the prior is strong.

“Extraordinary claims require extraordinary evidence”
- C Sagan

“the weight of evidence for an extraordinary claim must be proportioned to its strangeness”
- P Laplace

Strong priors require strong supporting evidence
Extraordinary claims require extraordinary evidence: the case of non-local perception, a classical and Bayesian review of evidences

Patrizio E. Tressoldi*

Dipartimento di Psicologia Generale, Università di Padova, Padova, Italy

Edited by:
Holmes Finch, Ball State University, USA

Reviewed by:
Jelle M. Wicherts, University of Amsterdam, Netherlands
Jeff Rouder, University of Missouri, USA

*Correspondence:
Patrizio E. Tressoldi, Dipartimento di Psicologia Generale, Università di Padova, Via Venezia, 8, 35131 Padova, Italy.
e-mail: patrizio.tressoldi@unipd.it

Starting from the famous phrase “extraordinary claims require extraordinary evidence,” we will present the evidence supporting the concept that human visual perception may have non-local properties, in other words, that it may operate beyond the space and time constraints of sensory organs, in order to discuss which criteria can be used to define evidence as extraordinary. This evidence has been obtained from seven databases which are related to six different protocols used to test the reality and the functioning of non-local perception, analyzed using both a frequentist and a new Bayesian meta-analysis statistical procedure. According to a frequentist meta-analysis, the null hypothesis can be rejected for all six protocols even if the effect sizes range from 0.007 to 0.28. According to Bayesian meta-analysis, the Bayes factors provides strong evidence to support the alternative hypothesis (H1) over the null hypothesis (H0), but only for three out of the six protocols. We will discuss whether quantitative psychology can contribute to defining the criteria for the acceptance of new scientific ideas in order to avoid the inconclusive controversies between supporters and opponents.

Keywords: meta-analysis, frequentist, Bayes, non-local perception
Posterior Probability = updated knowledge

Prior Distribution $\rightarrow$ Background Belief

$P(A \mid B) = \frac{P(A) \times P(B \mid A)}{P(B)}$

Likelihood Distribution $=$ Observational Data

Computational Complexity

Calculating the Marginal Likelihood can be computationally very intense

Sampling approximations like Monte Carlo models
Conclusion: Bayesian statistics

• Does not rely on the notion of a finding “as or more inconsistent with our H0” Frequentist approaches

• Starts with our understanding of how something works/ what is likely to happen

• We then update our belief based on our data

• Possible to perform multiple rounds of formulation of prior, evaluation of prior based on data and formulation of posterior.
\[ P(\text{I'm near the ocean} | \text{I picked up a seashell}) = \]
\[ \frac{P(\text{I picked up} | \text{I'm near}) P(\text{I'm near})}{P(\text{I picked up})} \]

Statistically speaking, if you pick up a seashell and don't hold it to your ear, you can probably hear the ocean.
Hackenberger BK. Bayes or not Bayes, is this the question? Croat Med J. 2019 Feb 28;60(1):50-52. doi: 10.3325/cmj.2019.60.50. PMID: 30825279; PMCID: PMC6406060.