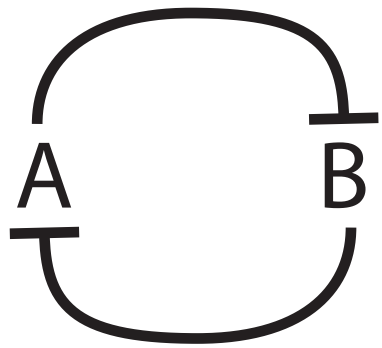
Question 1

Consider a network of two genes, A and B, that inhibit each others' production.



$$\frac{dA}{dt} = \frac{\rho}{1+B} - A$$

$$\frac{dB}{dt} = \frac{\rho}{1+A} - B$$

This is similar to the model we considered in class. The first term in each expression represents the maximal rate of production (ρ) divided by an inhibition term that includes the amount of the "other" species present. Note that in this model, we assume that A and B have the same maximal rate of production.

Write the equations that correspond to the nullclines for this system.

Show algebraically that equilibrium must occur where $A = B$.

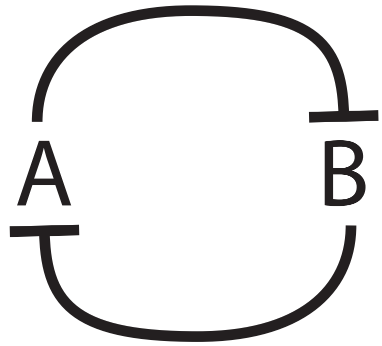
Solve for this equilibrium concentration in terms of ρ. Show your work.

Sketch the nullclines for this system. Label as many points as you can.

For this system, do you think it is possible for there to be more than one equilibrium point? Justify your answer.

Question 2

Consider a different network of two genes, A and B, that also inhibit each others' production. In this case, this model does not assume that the maximal levels of production are the same.



$$\frac{dA}{dt} = \frac{\alpha_0}{1+B} - A$$

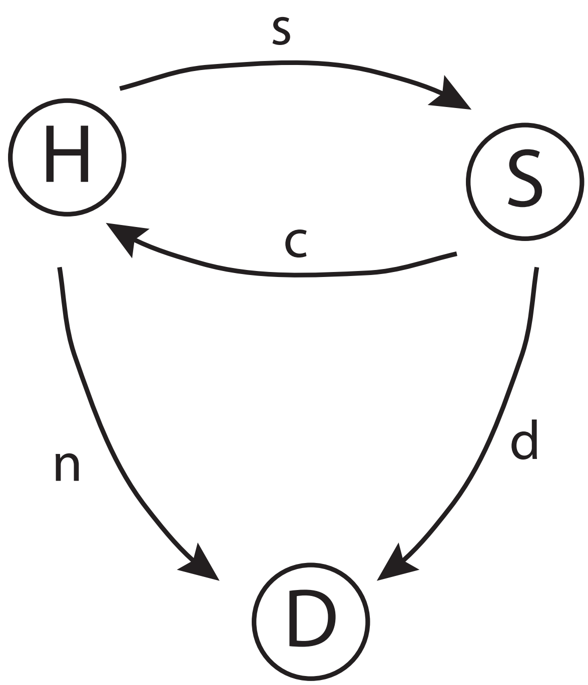
$$\frac{dB}{dt} = \frac{\beta_0}{1+A} - B$$

For this system, do you think it is possible for there to be more than one equilibrium point? If so, provide an example. If not, justify your reasoning.

Question 3

Your lab mate has heard about the wonders of Markov Models, and has been inspired to build one to represent the lifecycle of a river snail that she is studying. In a particular river, a bloom of algae causes snails to become infected at a high rate. In any given week, $s = 1/4$ of healthy snails become infected. Also, in a given week, $c = 3/4$ of infected snails recover, while the remaining $d = 1/4$ die. Finally, in a population of healthy snails, $n = 1/16$ will die in a given week.

Your lab mate has drawn the following diagram to specify a Markov model of this process.



In this diagram, the H state represents healthy snails, the S state represents infected (sick) snails, and D represents dead snails. All snails are taken to start their lives in the H state.

Is this model system ergodic? Explain your answer.

Write the difference equation and the transition matrix for this model.

Compute[1] how long snails live, on average.

[1] *Hint:* If $M = \left[\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right]$, then $M^{-1} = \begin{bmatrix} \frac{d}{(a\cdot d-b\cdot c)} & \frac{-b}{(a\cdot d-b\cdot c)} \\ \frac{-c}{(a\cdot d-b\cdot c)} & \frac{a}{(a\cdot d-b\cdot c)} \end{bmatrix}$

Further investigation has found that birds that feed on dead snails near this river are also dying. It has been suggested that this is due to birds eating snails that were infected when they died. **Compute the fraction of dead snails that were infected when they died.**

Question 4

An ion channel is modeled as a simple, two-state system with forward and reverse rate constants k_f and k_r:

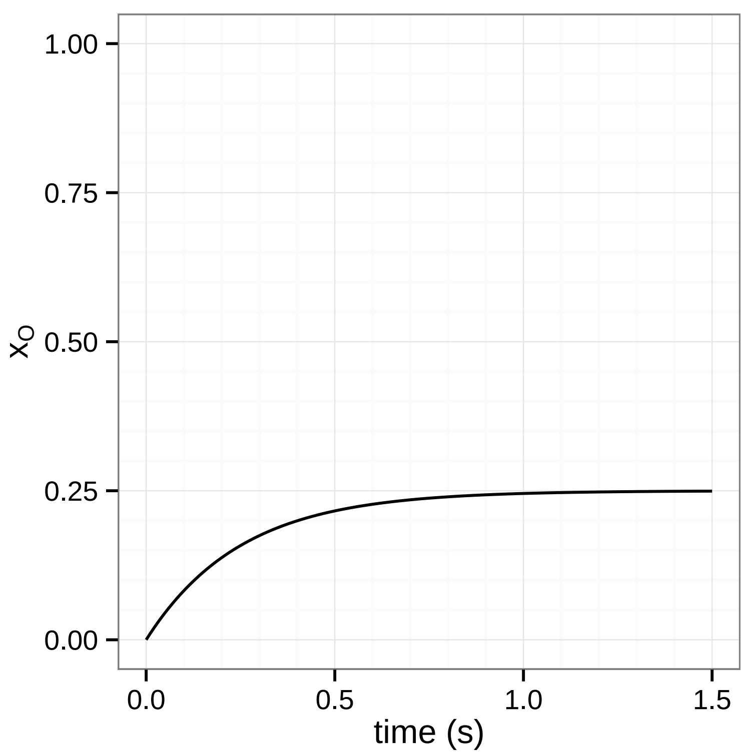
$$\mathrm{O} \underset{k_r}{\overset{k_f}{\rightleftarrows}} \mathrm{C}$$

$$\frac{dx_O}{dt} = -k_f x_O + k_r x_C$$

$$\frac{dx_C}{dt} = k_f x_O - k_r x_C$$

where x_C and x_O are the fractions of closed and open channels, respectively.

In a system that starts initially with all channels closed, the following curve has been measured showing the time-course towards the establishment of an equilibrium of the population.

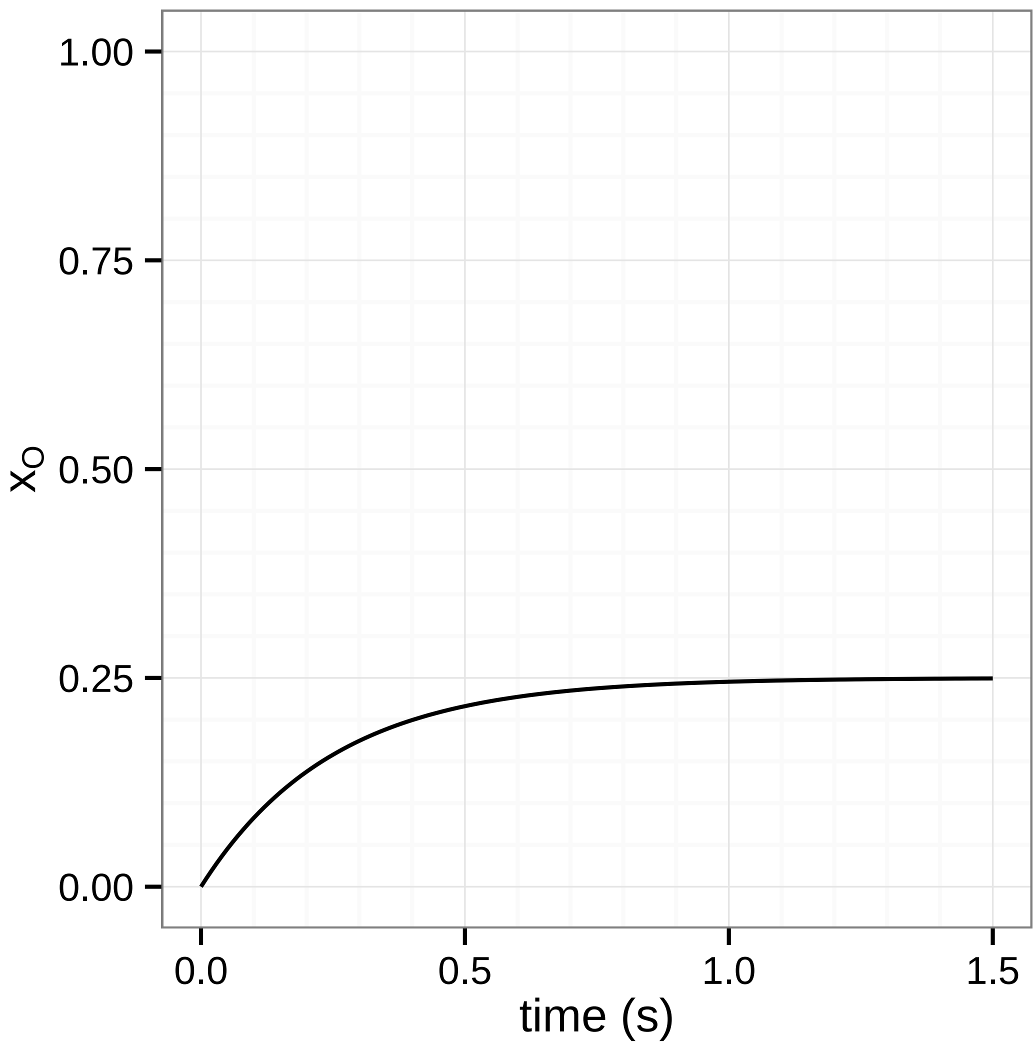


Estimate the forward rate constant, the reverse rate constant, and the equilibrium constant for this process. Explain your answer.

Estimate the time constant, τ, for this process.

Consider a different experiment conducted on the same system, which is started at $x_O =$ 0.75. **What would the time constant, τ be for this experiment?**

Carefully sketch the curve you would expect for the second experiment described above. Be as accurate as you can, and explain your reasoning/procedure.



[extra space if you need it]

[extra space if you need it]

[extra space if you need it]

[extra graph paper if you need it]

[extra graph paper if you need it]

[extra graph paper if you need it]