Name:

(please write your name on all pages)

Question 1

Consider a gene product that catalyzes its own formation with kinetics that follow a cooperative, Hill-like model, and for which the rate of degradation follows simple Michaelis-Menten kinetics.

$$\frac{dp}{dt} = \underbrace{\frac{\alpha \cdot p^2}{K_A^2 + p^2}}_{\substack{\uparrow \\ rate\ of \\ formation}} - \underbrace{\frac{\beta \cdot p}{K_B + p}}_{\substack{\uparrow \\ rate\ of \\ degradation}}$$

A) Sketch a plot of the rate of formation as a function of *p*. Be as precise as you can (i.e., label as many points, slopes and asymptotes as you can).

B) On the same axes, plot the rate of degradation as a function of *p*. Again, be as precise as you can. Use positive values for degradation, so that the curves you draw will cross if/when the rate of formation is equal to the rate of degradation.

C) Will p=0 always be a stable equilibrium? Why or why not?

D) Under what circumstances will a system that follows this model reach a stable equilibrium for arbitrarily large starting values of p? Explain your reasoning.

E) Is it possible for a system that follows this model to have a stable equilibrium where p>0. Explain your reasoning?

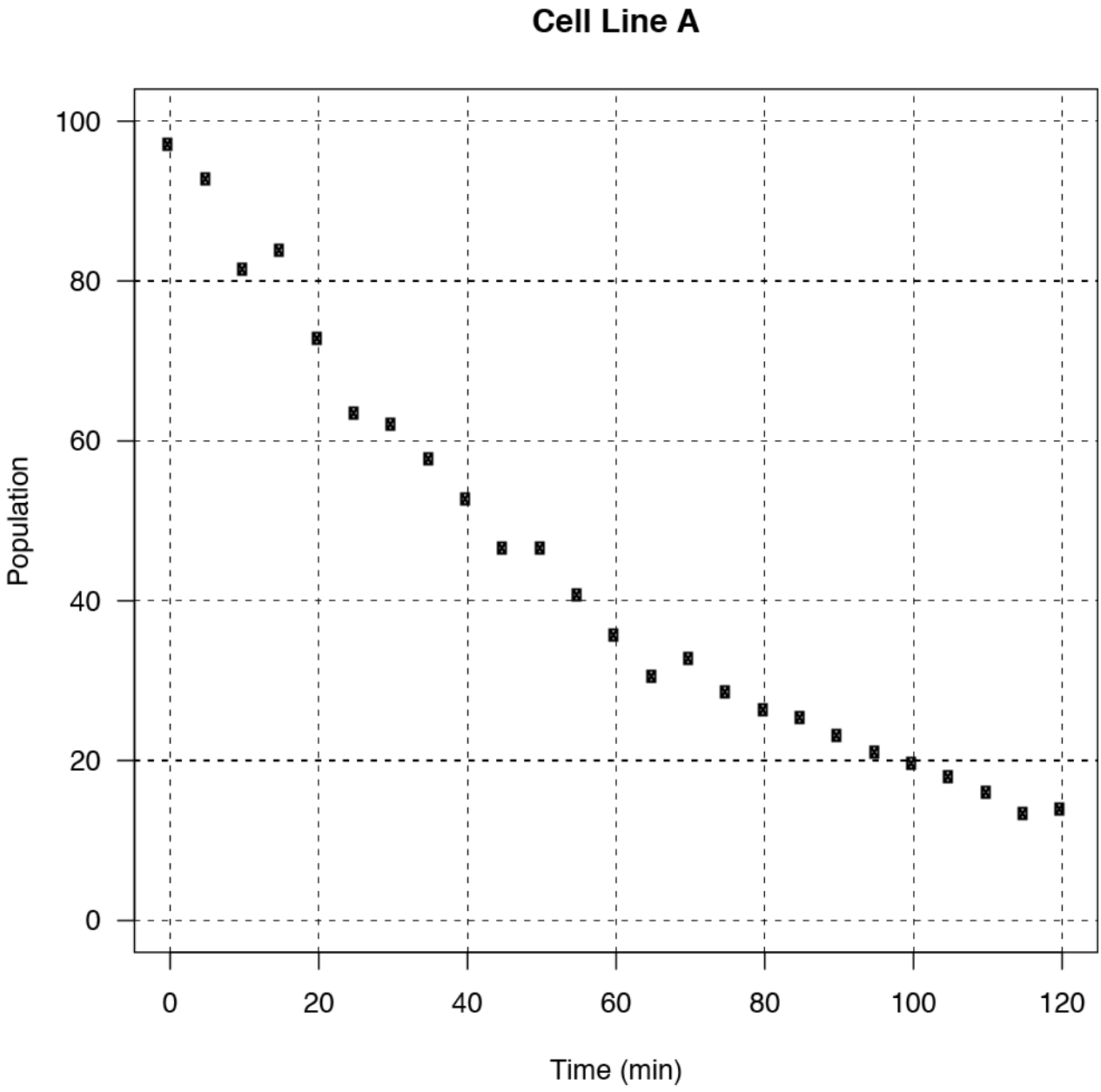
(for use in your explanations, if you need it)

(for use in your explanations, if you need it)

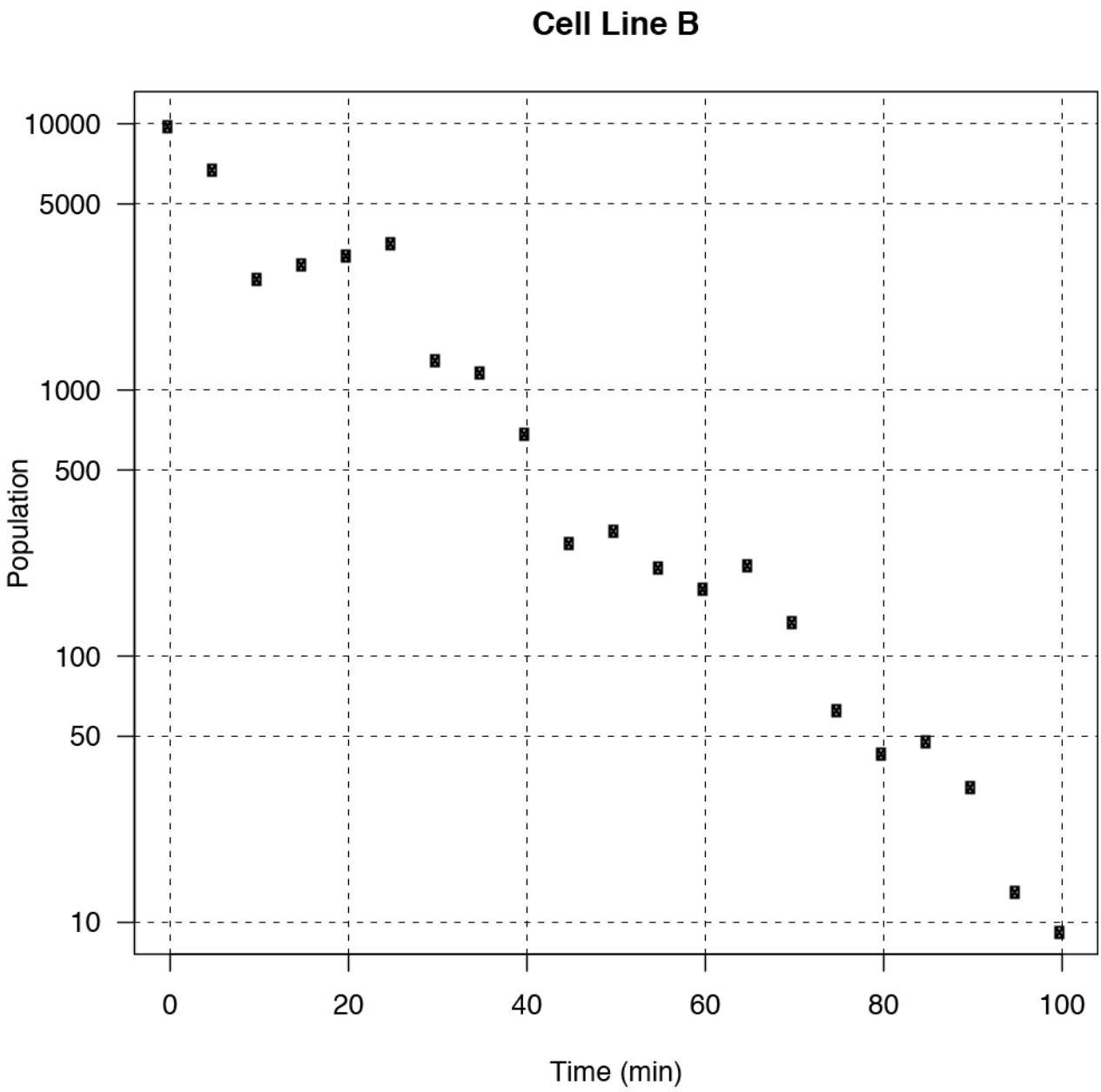
Question 2:

The figure below shows the time course of a cell population after exposure to a toxin that is known to be fatal to all cells.

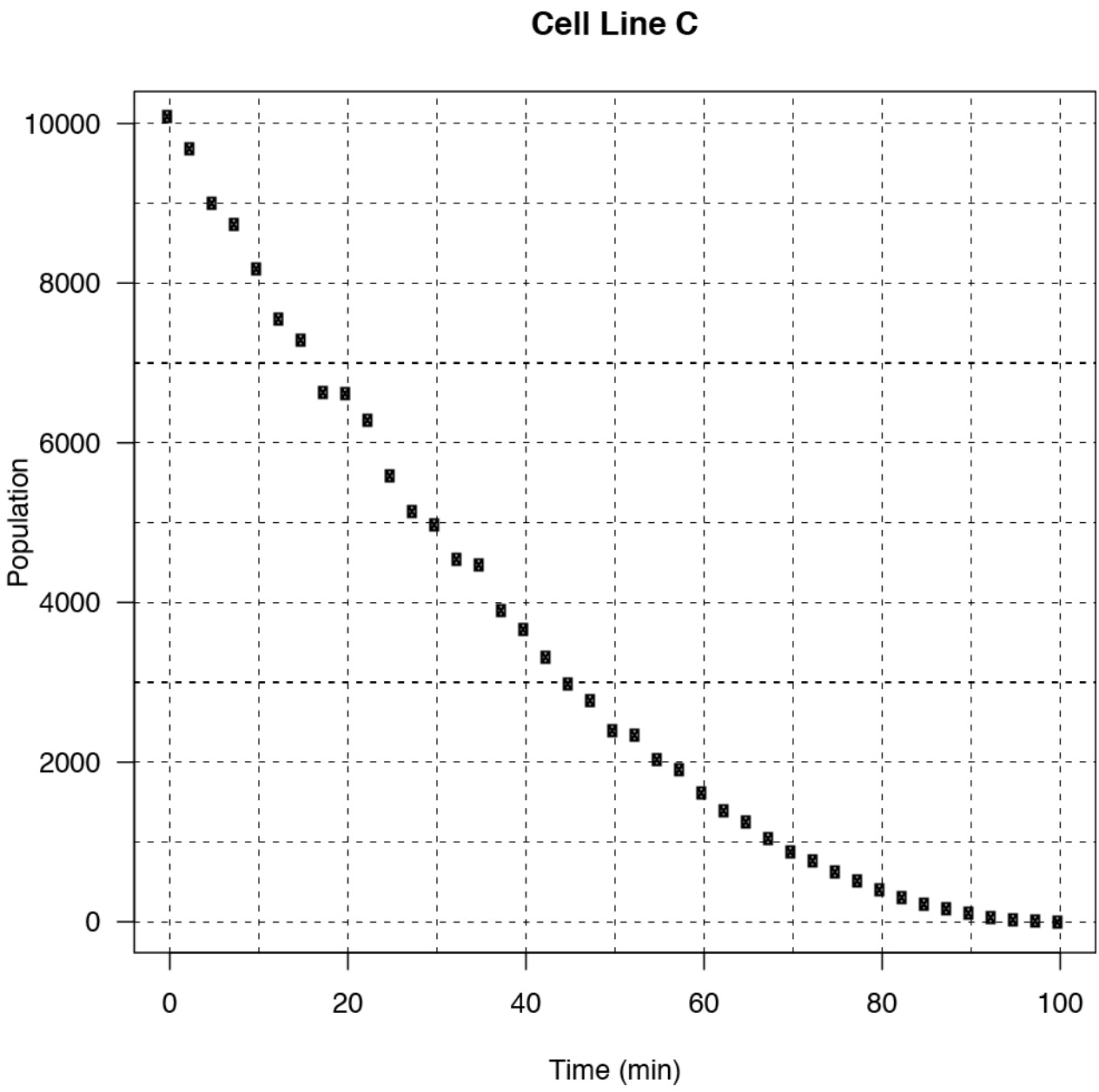
Estimate the time constant for the decay in cell population.



A well-respected collaborator of yours has obtained the data shown in the figure below for a similar experiment using a different cell line. **Estimate the time constant for the decay in cell population in this experiment.**



Another collaborator has shared the data in the figure below for an experiment from a third cell line. You are a little suspicious because the graduate student who reported the data was seen on the beach at the time he was supposed to be doing this experiment. **Can you estimate the time constant for the decay in cell population for this experiment?**



Question 3

Explain the phenomenon of aliasing, and what implications it has for determining sampling rate in experimental designs.

Your lab mate has performed an experiment that samples a waveform at 5 Hz. He is reporting a peak in the power spectrum at 1 Hz. You know that the phenomenon he is measuring should exhibit an oscillation at just above 30 Hz, and you are concerned that his results may be affected by aliasing. Your lab mate replies: "This is not a problem, because I also did the experiment with a 10 Hz sampling rate, and I see the same peak in the power spectrum. Since the power spectrum is converged, it must be OK. And besides, 5 Hz sampling is well over the limit needed to detect 1 Hz oscillations." **How would you respond?**

Question 4*

The Acme Precision Analytics Company is about to announce a new instrument for measuring the mass of a single cell. The details are a trade secret, but they claim that by careful analysis of a cell image using "hyperbolic third-order convolutions of perimeter image density gradients," one can accurately infer the mass of the cell. They claim that their method works for model cancer cells, and even small cells like red blood cells. Furthermore, they claim a stunning correlation coefficient of 0.99 between their measurements and the true values, and show you validation data to prove it…

```
> mass.validation.df
  measured.mass.pg true.mass.pg cell.type
1             3937          3836     H1650
2             4018          4097     H1650
3             3916          4148     H1650
4             4160          4115     H1650
5             4033          3939     H1650
6               27            45       RBC
> cor.test(mass.validation.df$measured.mass,
+          mass.validation.df$true.mass.pg)

      Pearson's product-moment correlation

data:  mass.validation.df$measured.mass and mass.validation.df$true.mass.pg
t = 25.74, df = 4, p-value = 1.354e-05
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 0.9715 0.9997
sample estimates:
  cor
0.997
```

* But see (after the exam):
Kevin G. Phillips, Steven L. Jacques, and Owen J. T. McCarty, *Phys. Rev. Lett.* 109, 118105; 2012.

Since you work in a highly regarded, Ivy League research lab, Acme is offering a free instrument for your lab in exchange for your endorsement. **How would you respond? Explain your reasoning.**

Bonus Question

Approximately how many base pairs are there in the human genome?

(extra space if you need it)

(extra space if you need it)

(extra space if you need it)

(extra space if you need it)