Name:

(please write your name on <u>all</u> pages)

Question 1

Consider an exogenous compound, A, which degrades according to a simple exponential:

$$\frac{d[A]}{dt} = -\alpha \cdot [A]$$

This compound catalyzes the formation of B, which itself degrades according to a simple exponential:

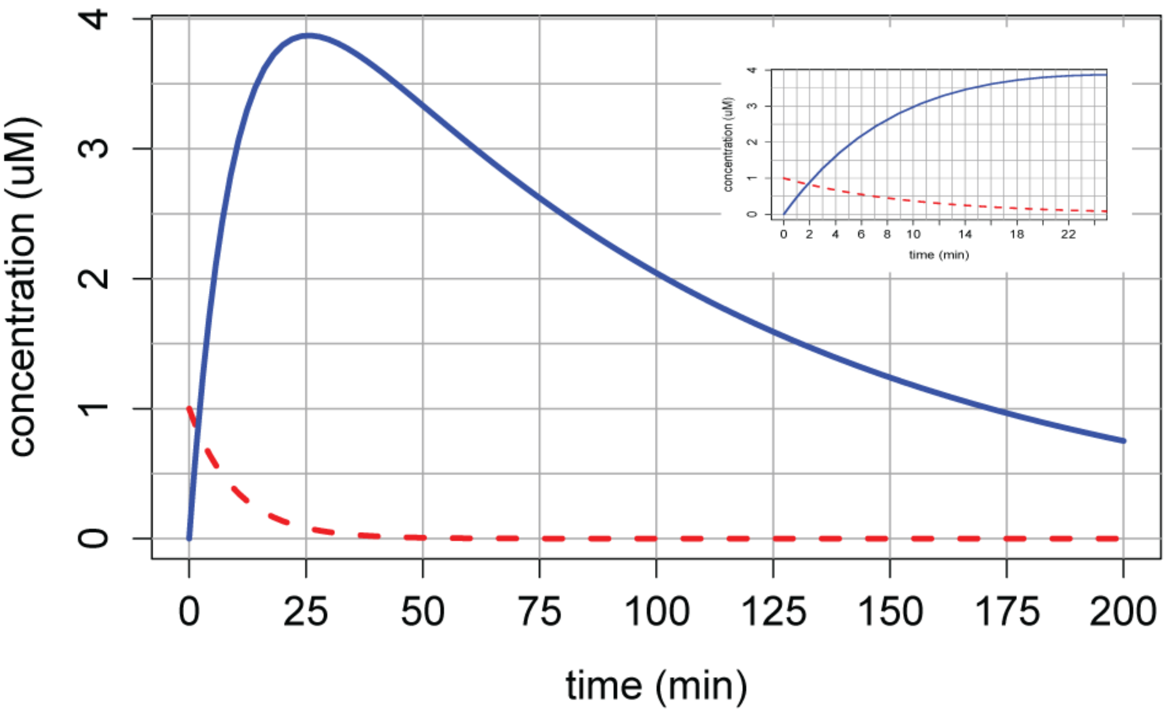
$$\frac{d[B]}{dt} = -\beta \cdot [B] + \gamma \cdot [A]$$

A) Write a differential equation in matrix form that represents this system.

B) Without any algebraic manipulation or computation, state one of the eigenvalues of the matrix from your answer above. Explain your reasoning.

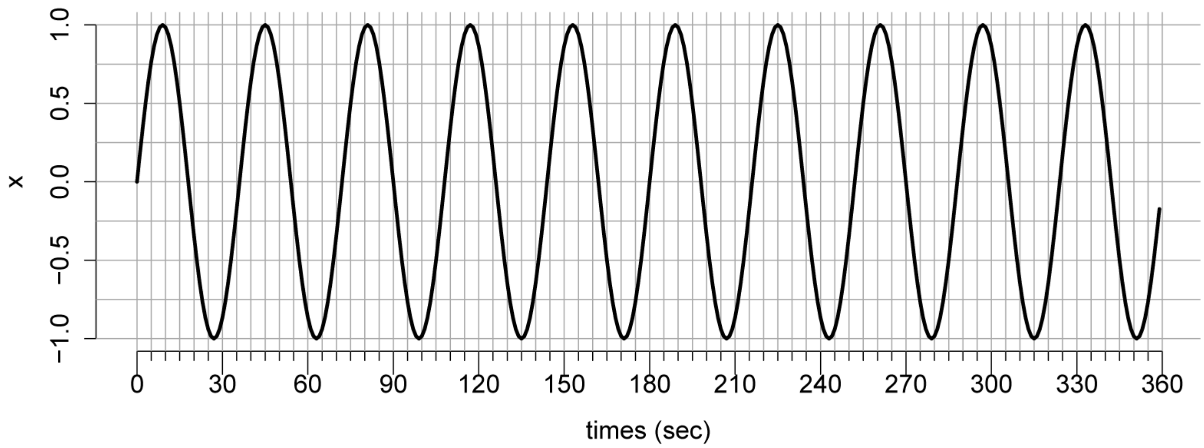
C) **What is the second eigenvalue? (use whatever techniques you like here)**

D) From the traces below, estimate the parameters α, β, and γ. Explain your reasoning.



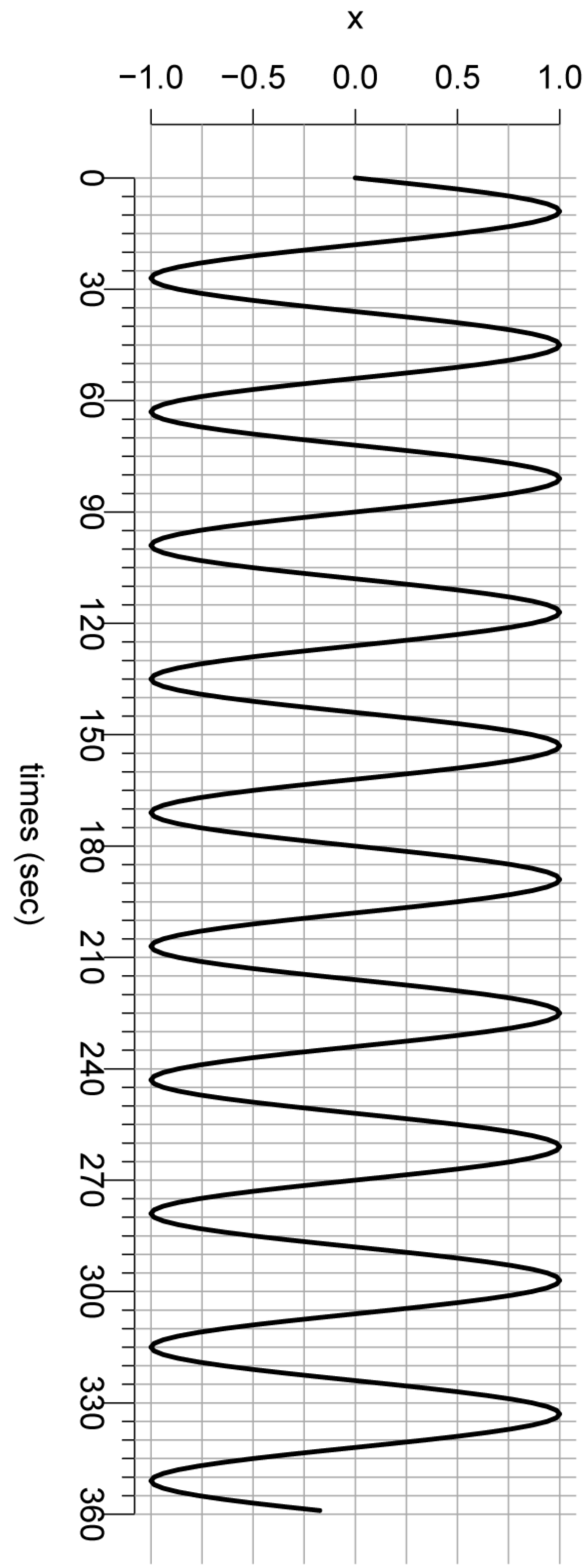
Question 2

The plot below shows a recording of a continuous signal.



A) What is the period of oscillation of the signal?

B) Mark the points that would be recorded if you sampled this signal every 30 seconds.



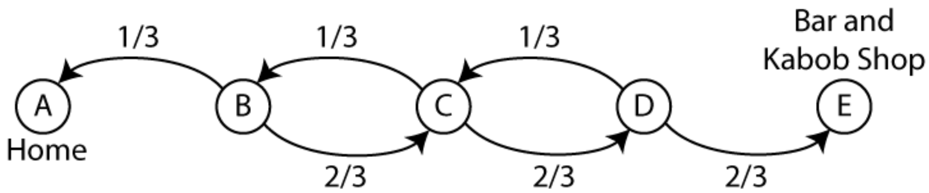
C) What is the apparent period of the sampled signal?

D) What is the minimum frequency at which you need to collect samples in order to detect the correct period of the underlying signal?

Question 3

A) What is the Markov property?

In class we considered a Markov chain that modeled a drunkard's walk, where there was a biased probably of moving to the right due to the odors emanating from a kabob shop:



Now imagine that there is a very scary looking mugger hidden at location D. Initially, our drunkard is unaware of the mugger's presence, but upon arriving at location D, he will become concerned for his safety, and in all subsequent moves he will be half as likely as before to move in the direction of the mugger.

B) If possible, construct a Markov chain that models this more elaborate scenario. Explain your reasoning. *(Hint: a given location may be represented by more than one state)*

Question 4

Consider a gene product that catalyzes its own formation according to Michaelis-Menten kinetics, and whose rate of degradation follows a simple exponential decay:

$$\frac{dp}{dt} = \underset{\substack{\uparrow \\ rate\ of \\ formation}}{\frac{V_{max} \cdot p}{K + p}} - \underset{\substack{\uparrow \\ rate\ of \\ degradation}}{\beta \cdot p}$$

A) Sketch a plot of the rate of formation as a function of *p*. Be as precise as you can (i.e., label as many points, slopes and asymptotes as you can).

B) On the same axes, plot the rate of degradation as a function of *p*. Again be as precise as you can.

C) Comment on the number of potential equilibrium points this system can exhibit, for both large and small values of β. Prepare appropriate phase portraits. Can you quantify the cutoff between "small" and "large" values of β?

Consider a different gene product that also catalyzes its own formation, demonstrating cooperative Hill-like kinetics, and whose rate of degradation also follows a simple exponential decay:

$$\frac{dq}{dt} = \underset{\substack{\uparrow \\ rate\ of \\ formation}}{\frac{V_{max} \cdot q^2}{K^2 + q^2}} - \underset{\substack{\uparrow \\ rate\ of \\ degradation}}{\beta \cdot q}$$

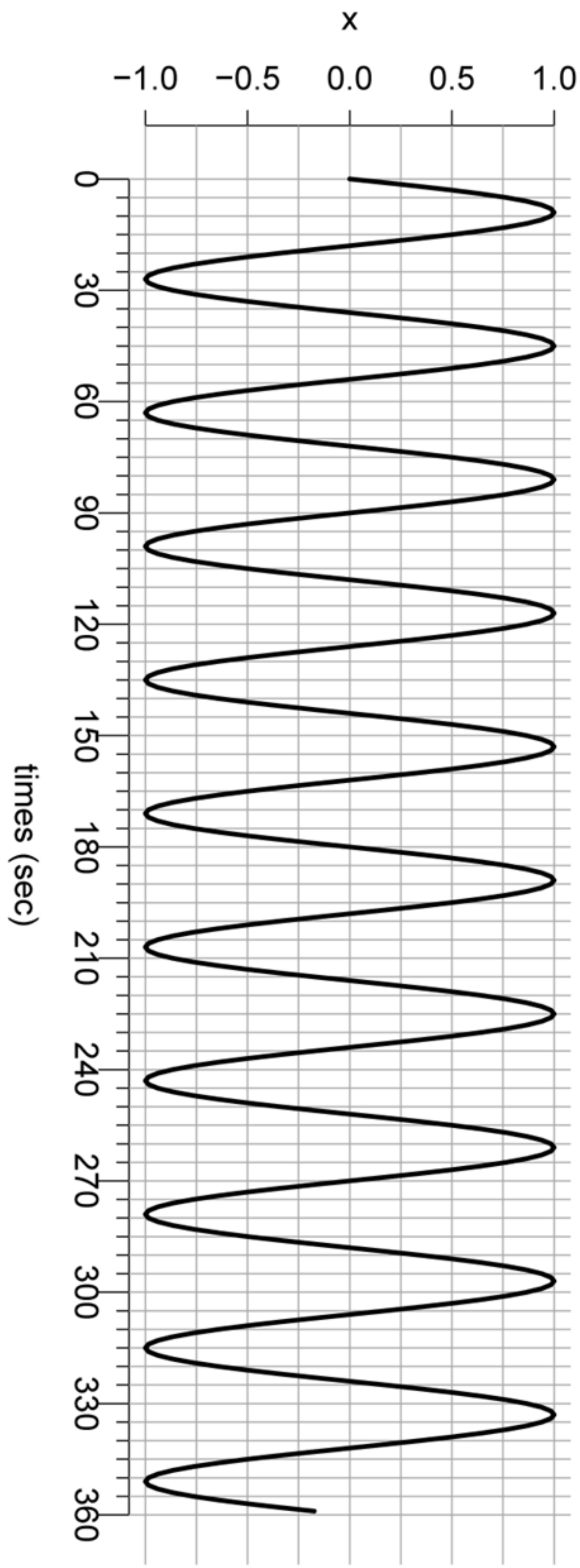
D) Sketch a plot of the rate of formation as a function of *q*. As usual, be as precise as you can.

E) On the same axes, plot the rate of degradation as a function of *q*. Again, be as precise as you can.

F) Comment on the number of potential equilibrium points this system can exhibit, for both large and small values of β. Prepare appropriate phase portraits. *(Don't worry about quantifying the cutoff between "small" and "large" values of β.)*

Name:

(extra copy of the signal in question 2 – just in case you need it)



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