Quantitative Understanding in Biology Module V: Fourier Series and PDEs Computer Laboratory

Fourier Series

The purpose of this laboratory is to explore the behavior of truncated Fourier series. Consider a pulse function, which we can model in Matlab with the commands…

```
x = zeros(100,1); x(45:55)=1; subplot(1,3,1); plot(x)
```

If we take the Fast Fourier transform of the sampled data and then de-transform it, we get the original function back. Note that due to rounding errors the results of `ifft` will contain small imaginary components; we can ignore them by explicitly taking only the real components of the result.

```
f1 = fft(x); subplot(1,3,2); plot(real(ifft(f1)))
```

If we truncate the Fourier series (by eliminating some of the high-frequency terms), and then de-transform the truncated series, we get an approximation to the original function. Recall that in Matlab's FFT output, the high frequency terms are in the middle of the returned array.

```
f2 = fft(x); f2(45:56)=0; subplot(1,3,3); plot(real(ifft(f2)))
```

Note the 'ringing' phenomenon near the discontinuities in the original function.

Experiment a bit with this technique. Try truncating fewer or more terms; get a feel for how the accuracy of the reconstituted signal varies with the number of preserved Fourier terms.

Try this for a few other functions (perhaps a sawtooth, or a Gaussian, or whatever you like)…

Try adding some noise to your functions, and then see how a reconstituted signal from a truncated series looks.

Partial Differential Equations

Work through the tutorial in the Matlab help for solving the Poisson equation. In the Matlab help, navigate to *Partial Differential Equation Toolbox->Getting Started->Solving a PDE*. Note that this documentation uses the symbol Δ for the Laplacian operator; we used ∇^2. Also, you'll probably want to read about Dirichlet and Neumann boundary conditions (Wikipedia is your friend here).

You don't need to hand anything in for this exercise, but you should complete the tutorial.

Challenge Problem (optional): Use the PDE toolbox to solve any biological problem of your choice.

