Experimental Power and Design

Quantitative Understanding in Biology

Tuesday, 17 September 2024

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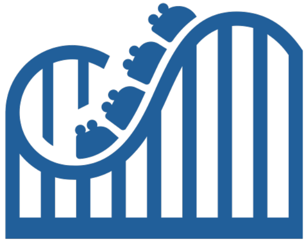
Review

Example 1

Middle school kids are going to the local state fair.



The school wants an estimate of the average height.

height

How many students should you measure if you want to get a good estimate of the true mean?

Let’s turn to the confidence interval!

Considering a single mean

Confidence Interval (CI)

$$CI: \bar{x} \pm t^{*} \cdot SEM$$

SEM = Standard Error of the Mean
$\bar{x}$ = Observed Mean

$$CI: \bar{x} \pm t^{*} \cdot \frac{SD}{\sqrt{n}}$$

For univariate distribution with large n

$$CI: \bar{x} \pm precision$$

We are defining precision as the half-width of the CI

Effect Size (d)

$$d = \frac{precision}{SD}$$

We are defining d as effect size

$$d < 1 \; hard \; to \; detect$$
$$d > 1 \; easy \; to \; detect$$

$$95\%\ CI\text{: } \bar{x} \pm 1.96 \frac{SD}{\sqrt{n}}$$

For univariate distribution with large n

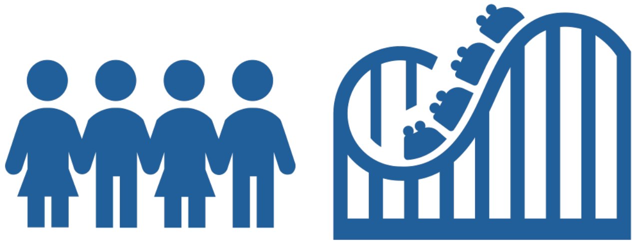
$$Let's\ approximate\ 1.96 \cong 2$$

$$precision = 2 \frac{SD}{\sqrt{n}}$$

$$\sqrt{n} \cdot precision = 2 \cdot SD$$

$$\sqrt{n} = 2 \cdot \frac{SD}{precision}$$

$$n \cong 4 \cdot \left(\frac{SD}{precision}\right)^2$$



$$95\%\ CI: \bar{x} \pm 1.96 \frac{SD}{\sqrt{n}}$$

For univariate distribution with large n

$$n \cong 4 \cdot \left(\frac{SD}{precision}\right)^2$$

$$SD = 5\ inches$$ (Can be based on previous data)

$$precision <= 2\ inches$$ (Based on what we define)

$$n \cong 4 \cdot \left(\frac{5}{2}\right)^2 = 25\ students$$

The above equations do NOT guarantee that if you perform n measurements, you will obtain the desired half-width.
In fact, you have ~50% chance of obtaining such a CI, or narrower.
Power is the probability of detecting the precision that you want to find.
In this case power is ~50%.

General Case

$$(1-\alpha)CI: \bar{x} \pm t^* \frac{SD}{\sqrt{n}}$$

For univariate distribution with large n

$$n \cong t^* \cdot \left(\frac{SD}{precision}\right)^2$$

Difference Between Two Means

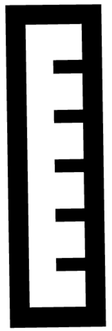
$$n_{each\ group} \cong 8\ \cdot \left(\frac{SD_{each\ group}}{precision}\right)^2$$

- Assumption: SDs of measurements from both groups are roughly the same.
- If not, use larger SD as general rule of thumb.
- Power ~ 0.5

Example 2

Group #1

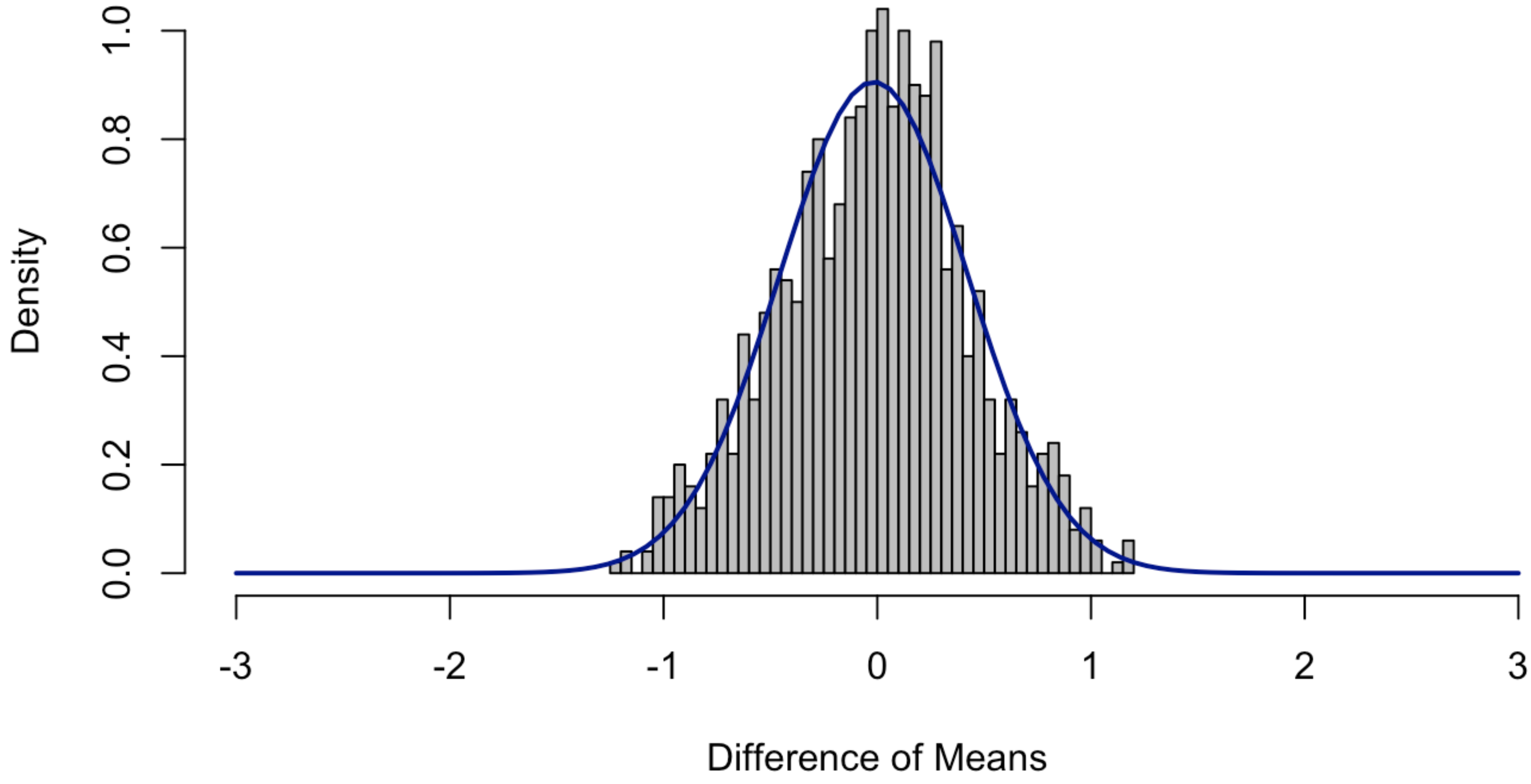
Group #2



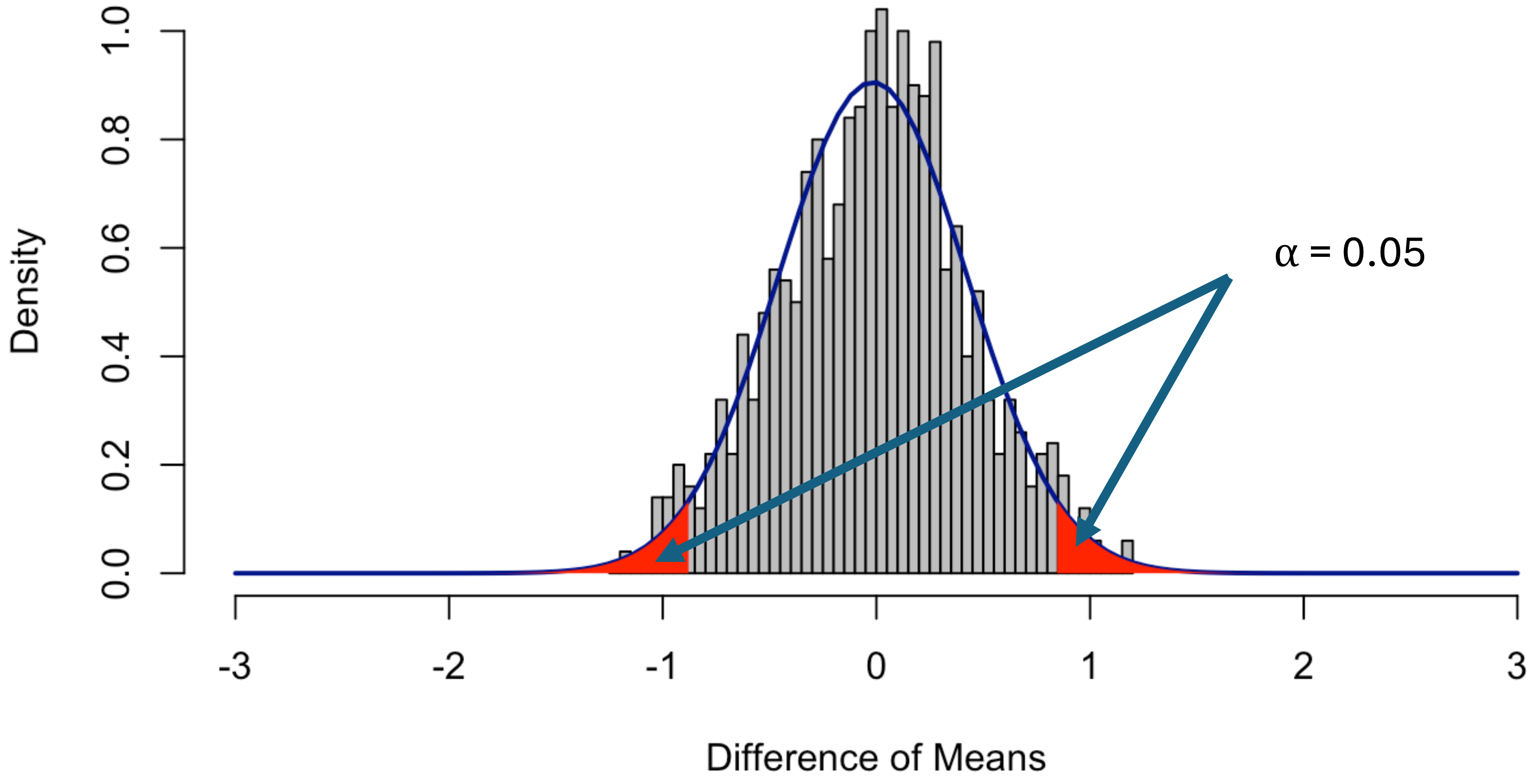
height

Simulate! → x_1 <- rnorm(10)
x_2 <- rnorm(10) → Difference of mean heights = $\bar{x}_2 - \bar{x}_1$ → Repeat x1000

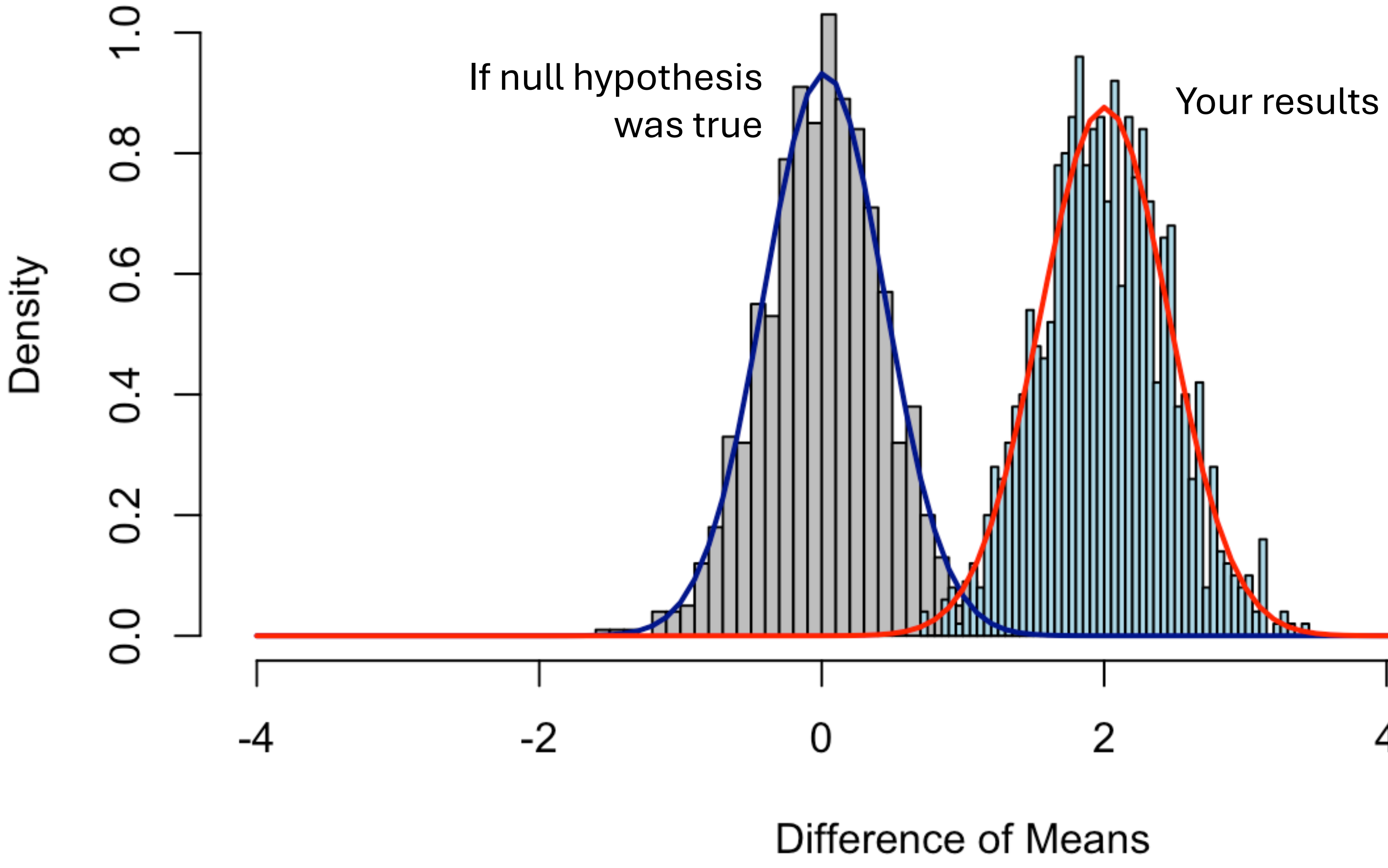
Simulate! → x_1 <- rnorm(10), x_2 <- rnorm(10) → Difference of mean heights = $\bar{x}_2 - \bar{x}_1$ → Repeat x1000



Type I Error (False Positive)



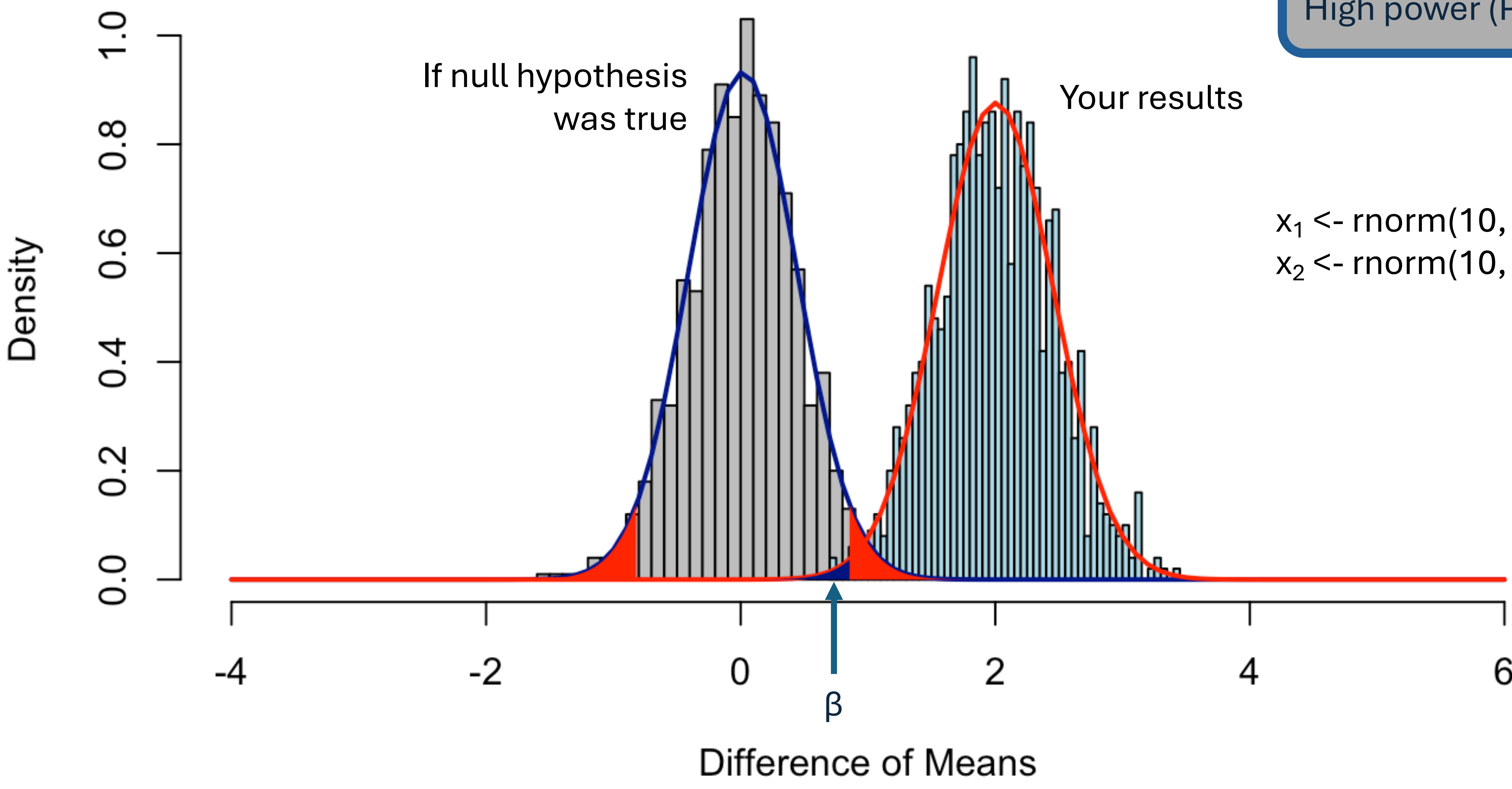
Type II Error: Case 1



x_1 <- rnorm(10, mean = 0, sd = 1)
x_2 <- rnorm(10, mean = 2, sd = 1)

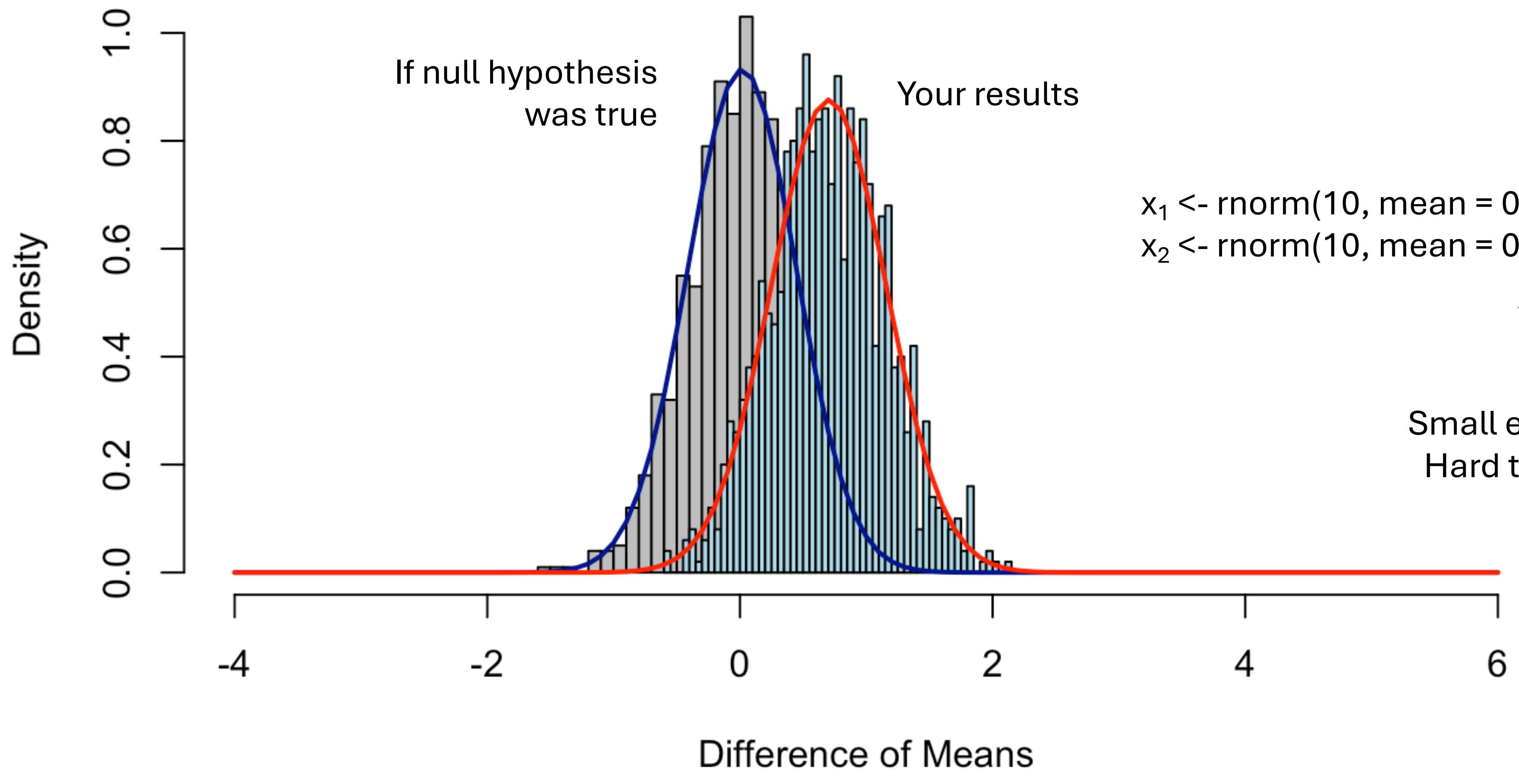
Type II Error (False Negative)

Small β
High power (Power = 1 – β)



x_1 <- rnorm(10, mean = 0, sd = 1)
x_2 <- rnorm(10, mean = 2, sd = 1)

Type II Error: Case 2

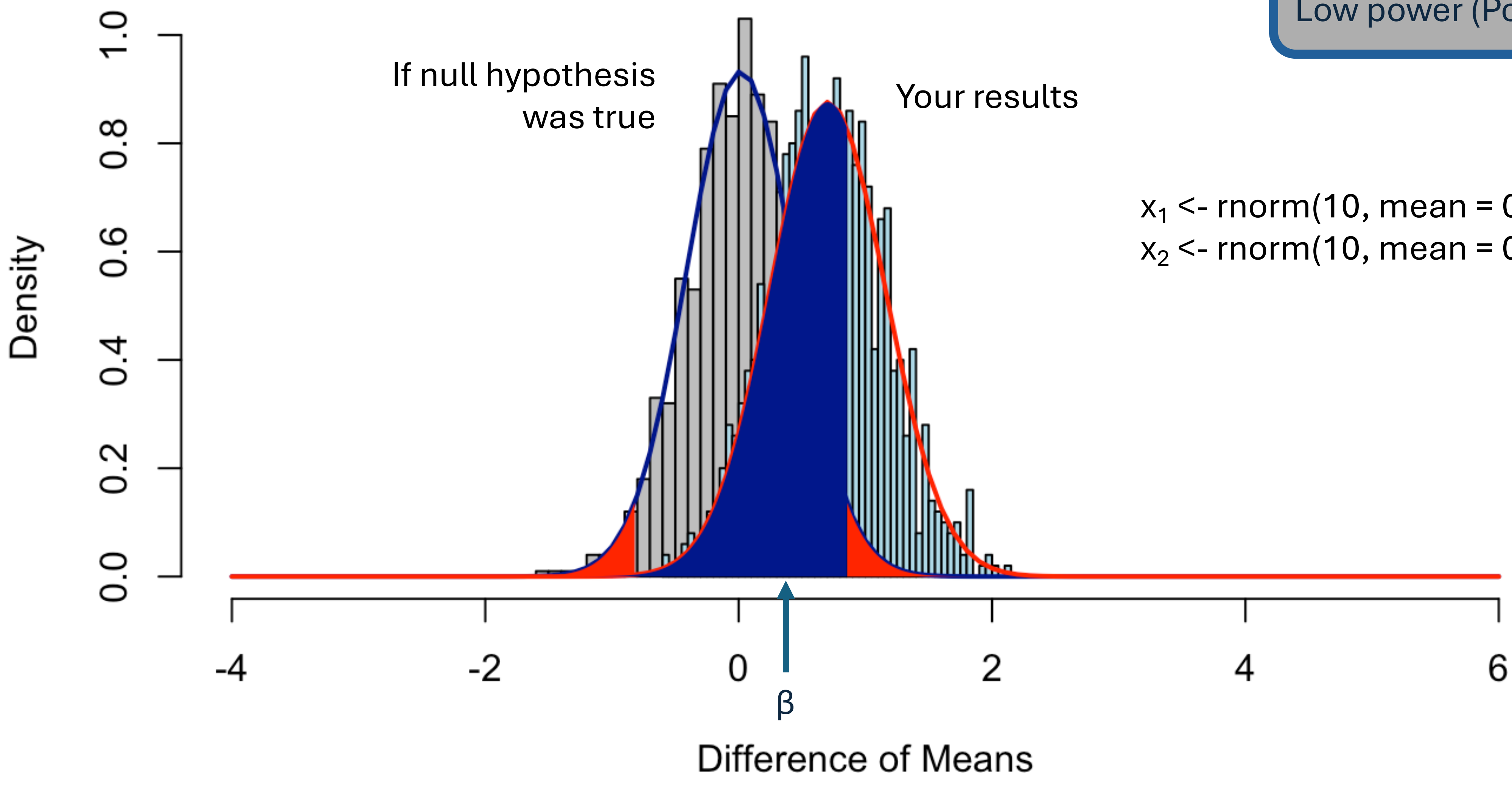


x$_1$ <- rnorm(10, mean = 0, sd =1)
x$_2$ <- rnorm(10, mean = 0.7, sd = 1)

Small effect size
Hard to detect

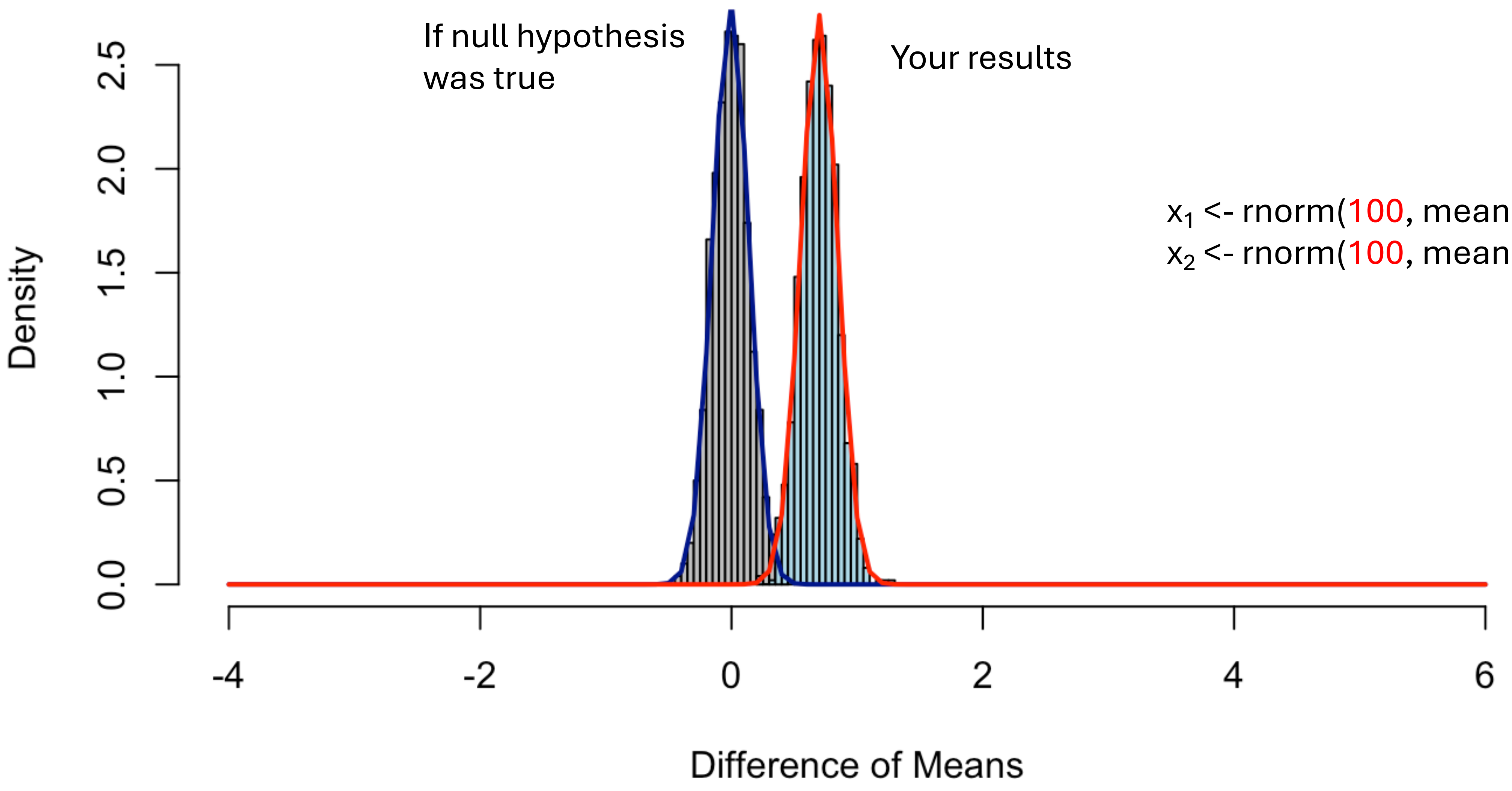
Type II Error: Case 2

Large β
Low power (Power = 1 – β)



x_1 <- rnorm(10, mean = 0, sd =1)
x_2 <- rnorm(10, mean = 0.7, sd = 1)

Type II Error: Case 3

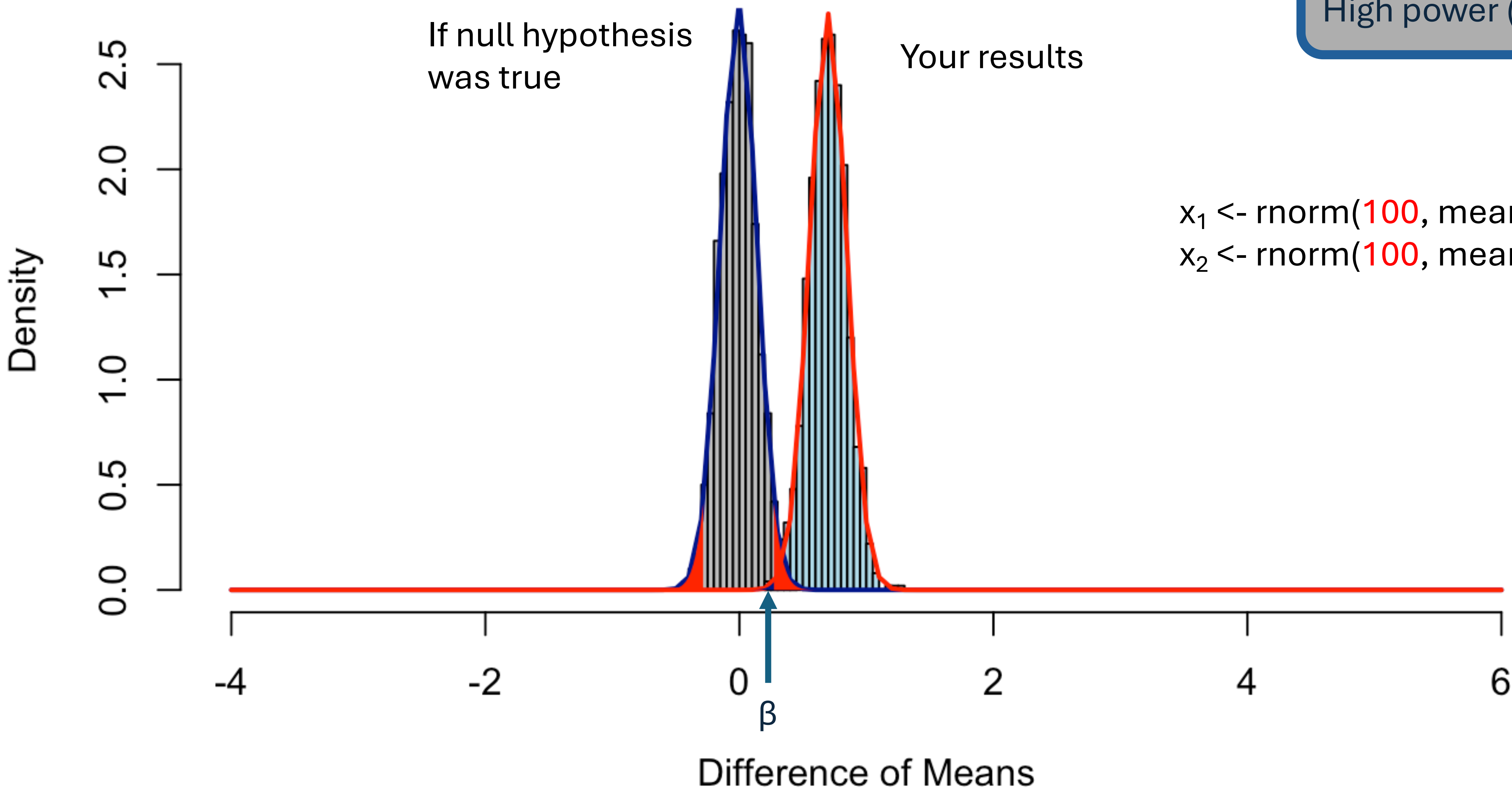


x_1 <- rnorm(100, mean = 0, sd =1)

x_2 <- rnorm(100, mean = 0.7, sd = 1)

Type II Error: Case 3

Small β
High power (Power = 1 – β)



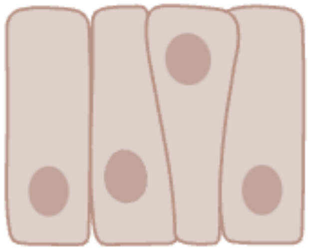
x_1 <- rnorm(100, mean = 0, sd =1)
x_2 <- rnorm(100, mean = 0.7, sd = 1)

Review Summary

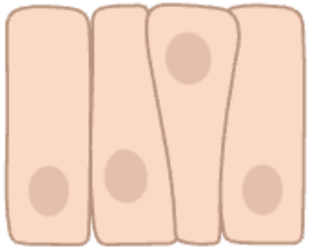
- Type I error (α):
  - Find a statistically significant difference when in fact there is none (False Positive)
- Type II error (β):
  - Do not find a statistically significant difference ***when in fact a difference between the two groups does exist*** (False Negative)
- Power (1-β):
  - Chance you'll be able to demonstrate an effect of a given size when it exists

Example 3

Wild-type



Knockdown



Preliminary Data: Transepithelial resistance (TER) Mean = 130 Ωcm² SD = 30 Ωcm²

If variations up to 35% in TER would be indicative of tight junction formation...

$$n_{each\ group} \cong 8 \cdot \left(\frac{SD_{each\ group}}{precision}\right)^2$$

$$= 8 \cdot \left(\frac{30\ \Omega\text{cm}^2}{45\ \Omega\text{cm}^2}\right)^2 = 3.5$$

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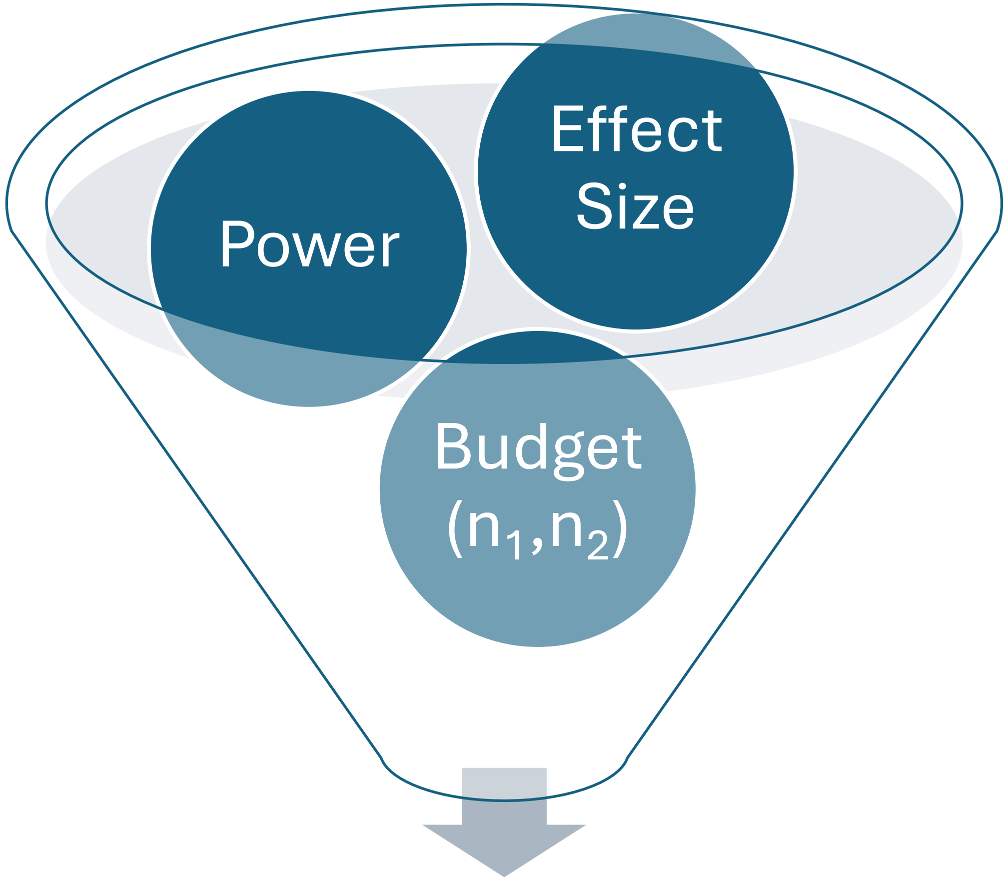
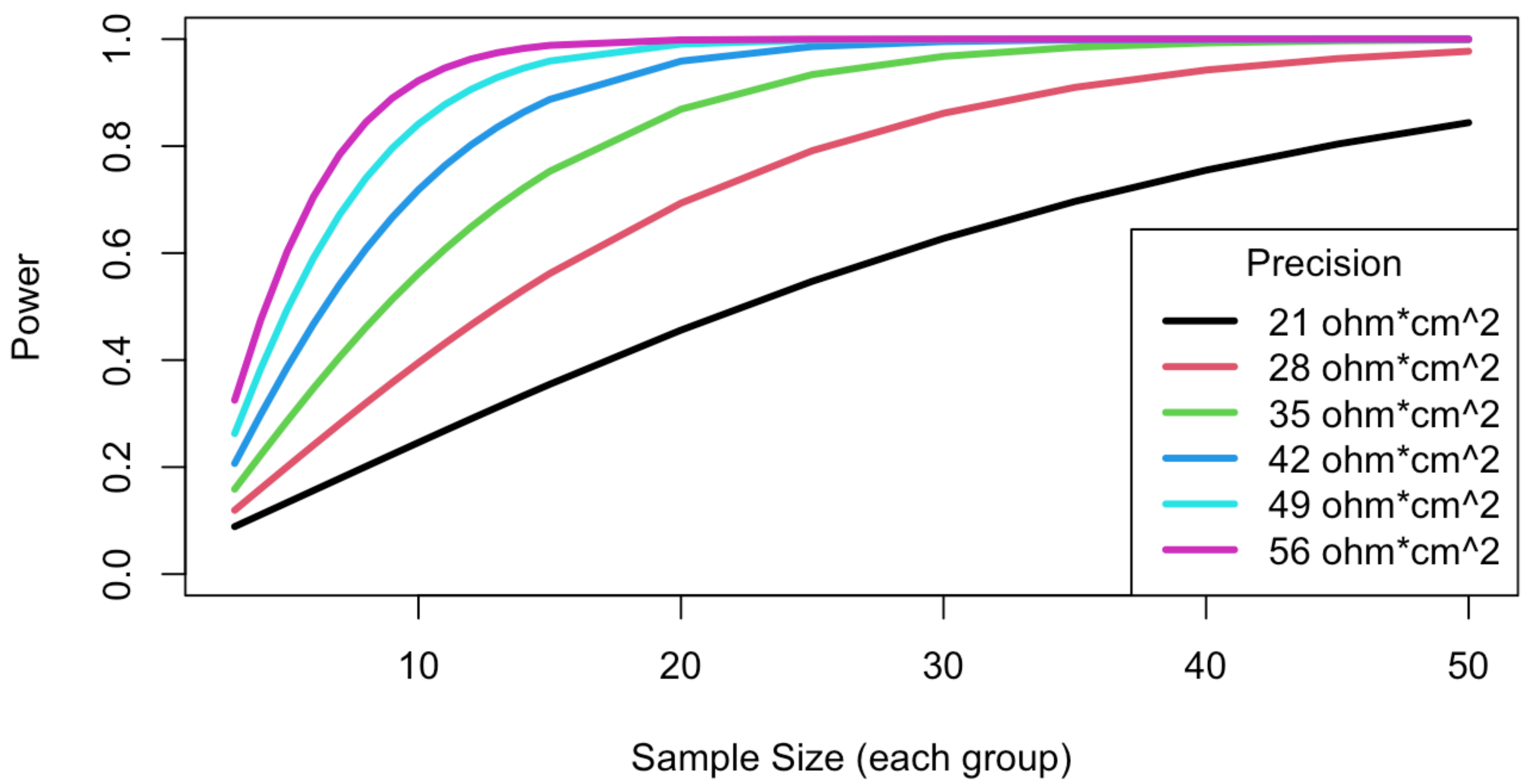
In practice ...



Experimental Design

Key Takeaways

- Experimental design considerations
  - Biological Question
  - Power
  - Resources
- Plan your experiment before conducting it
- Know the limitations of your experiment (i.e. Limited n)
- Statistical vs biological significance



Power
0.0
0.2
0.4
0.6
0.8
1.0
10
20
30
40
50
Sample Size (each group)
Precision
21 ohm*cm^2
28 ohm*cm^2
35 ohm*cm^2
42 ohm*cm^2
49 ohm*cm^2
56 ohm*cm^2