Question 1
In the presence of a toxin, a simple two-state (Open/Closed) channel can take on a third, permanently blocked state (B) when irreversibly bound to the toxin. When large amounts of toxin are present, this process can be represented by the Markov diagram below, where $\alpha$, $\beta$, and $\gamma$ are the transition probabilities for a 1 second time interval.

Is this system ergodic? Explain your answer.
This system can be modeled with the matrix equation

\[
\begin{bmatrix}
X_C \\
X_O \\
X_B
\end{bmatrix}_{n+1} = M \cdot \begin{bmatrix}
X_C \\
X_O \\
X_B
\end{bmatrix}_n
\]

Write the transition matrix \( M \):

What statements can you make about the eigenvalues of \( M \)?
If a population of such channels starts entirely in the Closed state at $t = 0$, and if $\alpha = 0.1$, $\beta = 0.1$, and $\gamma = 0.001$, prepare a sketch of $x_0(t)$. Be as accurate as you can, and explain your method.
Using the same parameters and initial conditions as above, prepare sketches of $x_c(t)$ and $x_o(t)$. Again, be as accurate as you can, and explain your method.

$\alpha = 0.1$, $\beta = 0.1$, $\gamma = 0.001$
Question 2
For the system described in Question 1, do you think a Markov representation would be appropriate if only a trace amount of the toxin were present? Explain your reasoning.
Question 3
Consider a protein where the active monomeric form enhances production of itself. This protein irreversibly dimerizes into an inactive form which is targeted for degradation. One way to model the dynamics of the active form is with the differential equation

$$\frac{dp_A}{dt} = -p_A^2 + \alpha_0 + \alpha \cdot p_A$$

Prepare a sketch of $\frac{dp_A}{dt}$ as a function of $p_A$. 
How many equilibrium points does this system exhibit? Discuss the stability of each.

What biochemical process might each of the three terms in the above model represent? For each term, discuss whether you think it is a reasonable mathematical model of that process.
Question 4
Through the semester, we saw at least two machine learning classification techniques: adaptive boosting (AdaBoost) and SVMs.

Briefly describe how SVMs work.

Briefly describe how adaptive boosting works.
What does it mean for a classification method to be an “unsupervised learning” method? Is adaptive boosting a supervised or unsupervised method? What about SVMs?

What does it mean for a classification method to be a “binary classifier”? Is adaptive boosting a binary classification method? What about SVMs?

Are the results from SVMs deterministic (i.e., if you train a classifier twice using the same data and parameters, will you get exactly the same classifier)? Is adaptive boosting deterministic? Explain.
Name:

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Name:

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