Question 1

A) The figure below shows the results of a numerical solution to a linear differential equation in one variable.

![Graph showing a declining exponential curve with time on the x-axis and value on the y-axis.]

Estimate the half-life of the process being modeled?

Estimate the time constant of the process being modeled?
B) The figure below shows the results of a numerical solution to a two-variable system of linear differential equations. The trace for only one variable is shown; the inset zooms in on the initial period of the simulation.

Estimate the fast time constant for the simulated process.

Estimate the slow time constant for the simulated process.
Would a plot of the second variable in the system necessarily show a similar rise and decay? Explain why or why not?
Question 2
Consider a system where the mRNA product of a gene inhibits its own production.

We can model the dynamics of this system with the differential equation...

\[
\frac{dm_A}{dt} = -m_A + \frac{\alpha_0}{1 + \beta m_A}
\]

...where the first term represents degradation of the mRNA and the second term represents its production. The maximum rate of production is \(\alpha_0\), and \(\beta\) is a parameter that indicates the strength of the inhibition.

Can this system ever have an unstable equilibrium? Explain why or why not.

Reminder:

\[
\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}
\]

The roots of \(ax^2 + bx + c = 0\) are \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\)
Can this system ever exhibit oscillatory behavior? Explain why or why not?
Question 3

A) What does it mean for a system to be ergodic?

B) What is the ergodic hypothesis?
C) Which of the following Markov chains are ergodic? Explain your reasoning.

Chain 1

Chain 2

Chain 3
Question 4

Your lab mate is looking for epigenomic markers which can be used to classify cancers. She performs a PCA analysis on the data, and presents the figure below in lab meeting, showing the first two principal components (PCs) for each case labeled by cancer type.

![PCA figure]

Your lab mate mentions that she is planning on trying to use an SVM (support vector machine) algorithm on these two PCs.

How well do you expect your lab mate’s SVM will be able to differentiate among these cancer types? Explain your reasoning.
How would you suggest your lab mate assess if the SVM is a good classifier?

What other observations and advice would you offer to help your lab mate with this classification problem?
(no questions here; just some extra space)
(no questions here; just some extra space)